

A Report on Theoretical Problems in Stability and Memory-Induced Dynamics of Fractional Infection Models

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This report addresses key theoretical problems in the stability analysis and memory-induced dynamics of within-host infection models governed by Caputo fractional derivatives. While fractional-order formulations preserve the threshold parameter R_0 , the non-local memory kernel introduces distinct qualitative challenges to classical dynamical systems theory. First, we report on the problem of fractional Hopf-type bifurcations: because Caputo derivatives preclude genuine periodic orbits, crossing the angular threshold $|\arg(\lambda)| = q\pi/2$ generates long transient, quasi-periodic-like oscillations whose analytical decay rates and phase-space structures remain uncharacterized. Second, we report the limitations of Lyapunov methods in establishing global asymptotic stability; although constructing suitable Lyapunov functions confirms global Lyapunov stability ($D^q L(t) \leq 0$), the loss of the classical semi-flow property prevents direct application of LaSalle's invariance principle or Poincaré – Bendixson theory. This report details these specific analytical barriers to clarify the current theoretical boundaries between hereditary memory effects and asymptotic stability.