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PROXIMAL POINT ALGORITHM ON GEODESIC SPACES WITH A NEW RESOLVENT OPERATOR

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The proximal point algorithm is one of the most useful methods for finding a minimizer of a given convex function. It was introduced by Rockafellar [8] in the setting of Hilbert spaces. In this method, resolvents of a convex function play an important role in approximating minimizers of a given convex function. In a complete CAT(0) space X, a resolvent for a convex function f is defined by

$$J_f x = \operatorname*{argmin}_{y \in X} \left(f(y) + \frac{1}{2} d(y,x)^2 \right)$$

for $x \in X$, see [7]. Using this resolvent, Bačák [1] investigated the proximal point algorithm in complete CAT(0) spaces and proved that the sequence $\{J_{\lambda_n f} x_n\}$ is Δ-convergent to a minimizer of f. In 2016, Kimura and Kohsaka [2] introduced the following resolvent in an admissible complete CAT(1) space:

$$R_f x = \operatorname*{argmin}_{y \in X} \left(f(y) + \tan d(y,x) \sin d(y,x) \right)$$

for $x \in X$. On the other hand, Kimura and the present author [3] proposed another type of resolvent defined by

$$S_f x = \operatorname*{argmin}_{y \in X} \left(f(y) - \log(\cos d(y,x)) \right)$$

in an admissible complete CAT(1) space.

In 2024, Kimura and Sudo [5] established a fixed point theory on Banach spheres equipped with a convex structure. They also introduced a resolvent of a convex function f having a minimizer on a Banach sphere X by

$$R_f x = \operatorname*{argmin}_{y \in X} \left(f(y) - \langle y, Jx \rangle \right)$$

for $x \in X$, and investigated the proximal point algorithm associated with this resolvent, see [6].

In this talk, we investigate the proximal point algorithm generated by a new resolvent in geodesic spaces.

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