A FRAMEWORK OF SMOOTHING PENALTY METHODS FOR CONVEX SECOND-ORDER CONE PROGRAMMING: FUNDAMENTALS AND APPLICATION

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Penalty and barrier methods have been extensively developed as fundamental techniques in optimization theory. Against this background, primal-dual interior-point methods have emerged as highly efficient algorithms. Currently, many commercial solvers for Second-Order Cone Programming (SOCP) and Semidefinite Programming (SDP) rely on these interior-point methods, making them undeniably central to modern optimization applications. However, there remain specific scenarios where primal-dual interior-point methods struggle. In this presentation, we propose a novel smoothing penalty method utilizing asymptotic cones. We will also explore its practical applications to Model Predictive Control.

Here, we focus on the general convex second-order cone programming (CSOCP) problem. Let $\mathcal{K}$ denote the general second-order cone (SOC) in $\mathbb{R}^n$ given by

$$\mathcal{K} := \mathcal{K}^{n_1} \times \cdots \times \mathcal{K}^{n_r},$$

with $r, n_1, \ldots, n_r \geq 1$, $n_1 + \cdots + n_r = n$, and

$$\mathcal{K}^{n_i} := \left\{ (x_1, x_2) \in \mathbb{R} \times \mathbb{R}^{n_i - 1} \;\middle|\; \|x_2\| \leq x_1 \right\},$$

where $\|\cdot\|$ denotes the Euclidean norm. Note that $\mathcal{K}^1$ denotes the set of nonnegative real numbers $\mathbb{R}_+$. In this presentation, we consider the following CSOCP problem:

$$\begin{cases} \min & f(\xi) \\ \text{s.t.} & A\xi - b \preceq_{\mathcal{K}} 0, \end{cases}$$

where A is an $n \times m$ matrix with $n \geq m$ and $\operatorname{rank}(A) = m$, $b \in \mathbb{R}^n$, $f : \mathbb{R}^m \to \mathbb{R} \cup \{+\infty\}$ is a closed proper convex function, and $x \preceq_{\mathcal{K}} 0$ means $-x \in \mathcal{K}$.

For simplicity, we focus on a single second-order cone $\mathcal{K}^n$, as the analysis in this setting can be naturally extended to the general second-order cone $\mathcal{K}$. In other words, we primarily investigate the following formulation:

$$\begin{cases} \min & f(\xi) \\ \text{s.t.} & A\xi - b \preceq_{\mathcal{K}^n} 0. \end{cases}$$

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