NUMERICAL EXPERIMENTS FOR ESTIMATING SET INEQUALITIES VIA SUBLINEAR SET FUNCTIONALS WITH SETOPTTOOLS

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In set optimization, various definitions and applications of set inequalities (e.g., [1, 2]) have been studied. However, there are still few challenging or compromise-oriented applications to practical problems. Although several researchers have actively investigated set-valued objects against such issues (e.g., [3, 4, 5, 6]), the vast majority of the literature focuses on theoretical problems and formal extentions.

Some of the reasons lie in the difficulty of handling infinite dimensional spaces and unbounded sets, which should be solved finitely and algorithmically. Another problem is the shortage of publicly available resources, such as software packages or calculation modules. Recently, there are a lot of social services enough for such data to be easily published, however, it is still not easy to reproduce and verify their results computationally.

The author has impremented a calculation package for Python language ([7]) named "SetOptTools." This package includes computational functions for set relations between sets represented by systems of linear inequalities such as $\{\boldsymbol{x}|A\boldsymbol{x} \leq \boldsymbol{b}\}$. The system adopts sublinear characterization methods proposed in ([5]) to evaluate "set relations" defined in [1]. Moreover, characterization functionals have been studied to quantify set relations (e.g., [8, 9, 10, 11]). This research is motivated by characterization theorems (e.g., [8]), which establish the equivalence of set relations and the negativity of characterizing scalars.

For impremention, this module consists of four different subsystems. Each function requires the compared sets and a convex ordering cone represented as the intersection of finitely many half spaces. These subsystems independently scalarize set inequalities by checking the consistency and redundancy of given inequalities and the optimality of evaluation problems as follows.

The first one computes strict Type 1 set relations, which can be estimated by a series of linear maximization problems. The second one handles stronger Type 2U and 2L relations, which are obtained by a combination of linear maximization and minimization problems. On the other hand, pre-ordering Type 3U and 3L require several steps: decomposing the compared sets into a bounded region (convex hull) and unbounded directions (rays); checking some algebraic geometry related to the unbounded directions of the given sets and the cone; computing the exact values of characterization functionals. The last one deals with the weakest Type 4 relations by solving several linear programming problems.

In this talk, we first introduce SetOptTools and its technical concepts. Next, we consider some numerical examples of evaluating set inequalities using the package.

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