A CLOSED-FORM DISTRIBUTIONALLY ROBUST DECISION RULE UNDER CONFORMALLY CALIBRATED WASSERSTEIN AMBIGUITY SETS

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Distributionally robust decision-making requires uncertainty sets that are both statistically valid and operationally meaningful under demand uncertainty. We propose a feature-adaptive distributionally robust optimization framework in which the Wasserstein-2 ambiguity radius is calibrated using split conformal prediction [1, 2]. Specifically, the Wasserstein ambiguity radius is set equal to the split conformal quantile of calibration residuals, establishing a direct connection between finite-sample statistical validity and distributionally robust optimization [3].

Under a Gaussian reference distribution fitted via NGBoost [4] and an upper-tail shift approximation, the resulting formulation yields a tractable closed-form decision rule that decomposes into two structurally distinct components: a context-adaptive DRO adjustment and a conformal correction term. This decomposition reveals a separation of roles within the framework. Conformal calibration guarantees finite-sample coverage independently of distributional assumptions, while the DRO component provides a robustness-oriented interpretation under the Gaussian-reference approximation [5].

Several theoretical properties are established. We show that the finite-sample coverage guarantee is preserved under the proposed DRO construction, characterize how heavy-tailed demand attenuates sensitivity to the cost ratio, and derive an explicit measure quantifying the value of improved forecasting precision. The framework links statistical reliability, decision robustness, and predictive uncertainty within a unified optimization structure, following the prescriptive analytics paradigm of [6].

Empirical validation is conducted using 953,736 London Fire Brigade incident records. The proposed method adapts deployment decisions from 2.1 to 7.2 appliances depending on context, compared with a fixed deployment level of 4.1 under a conventional fixed-radius DRO approach [7], while reducing average deployment by approximately 20% at comparable coverage levels. Additional out-of-time validation on 304,858 incidents from 2015–2017 confirms that the coverage guarantee remains valid across a distinct historical period without parameter re-tuning.

References

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