# STRONG CONVERGENCE THEOREMS FOR ZERO POINTS OF MAXIMAL MONOTONE OPERATORS AND ITS APPLICATIONS

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In 2008, Takahashi, Takeuchi and Kubota [7] introduced the shrinking projection method which is an iterative method for finding a common fixed point of some families of nonlinear mappings in a Hilbert space. In 2009, Kimura and Takahashi [5] improved this method in a Banach space. In the shrinking projection method, we need to obtain the exact value of the metric projection to generate a sequence in every step, which is a difficult task. To solve this problem, in 2012, Kimura [4] proposed the shrinking projection method with nonsummable errors in a geodesic space. Motivated by Kimura [4], in 2019, Takeuchi [6] presented another method which is called the shrinking projection method with allowable ranges. They obtained a strong convergence theorem for finding a fixed point of a nonlinear mapping in a Banach space.

On the other hand, the shrinking projection method is also used to establish convergence theorems for finding a zero point. In 2009, by using the original shrinking projection method, Inoue, Takahashi and Zembayashi [3] obtained a strong convergence theorem for finding a zero point of a maximal monotone operator in a Banach space. In 2016, Ibaraki [1] studied the shrinking projection method with nonsummable errors introduced by Kimura [4] for the zero point problem in a Banach space. In 2025, Ibaraki and Kajiba [2] studied the shrinking projection method with allowable ranges introduced by Takeuchi [6] for the zero point problem in a Banach space.

In this talk, we study the shrinking projection method with allowable ranges by using the generalized projection for the zero point problem. The generalized projection is a different type of projection from that used by Ibaraki and Kajiba [2]. Using this method, we obtain strong convergence theorems for finding a zero point of a maximal monotone operator in a Banach space. Moreover, using our results, we consider several applications such as the convex minimization problem and the variational inequality problem.

## References

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