Taiwan-Japan Joint Workshop on Nonlinear Analysis and Optimization (TJWNAO2026)
held in Nagaoka Univ. on Aug.3 and Niigata Univ. on Aug.4–5, 2026

NEW SET-VALUED EKELAND'S VARIATIONAL PRINCIPLE FOR ESSENTIAL DISTANCES WITH APPLICATIONS

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The celebrated Ekeland's variational principle (see [5, 6]), established in 1972 for characterizing approximate solutions to nonconvex minimization problems in metric spaces, has emerged as a cornerstone of nonlinear functional analysis and applied mathematics. Benefiting from its profound theoretical significance and versatility, Ekeland's variational principle has spurred extensive research into its generalizations in diverse directions; see, for instance, [1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16] and the references therein. It is well known that the primitive Ekeland's variational principle is equivalent to the Caristi's fixed point theorem and to the Takahashi's nonconvex minimization theorem (for more details, see, e.g., [9, 11, 12, 14, 16]).

In this talk, we present new versions of the generalized Ekeland's variational principle for set-valued mappings and essential distances (see, e.g., [3, 4]) in the setting of (L, K)-lower complete metric spaces (see, e.g., [14]). These results are then applied to establish some new Caristi type fixed point theorems, Takahashi type nonconvex minimization theorems and others.

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