SHRINKING PROJECTION METHODS WITH EXTENDED ALLOWABLE RANGES FOR A FAMILY OF QUASI-NONEXPANSIVE TYPE MAPPINGS

TAKANORI IBARAKI
YOKOHAMA NATIONAL UNIVERSITY IN JAPAN

Fixed point approximation methods for nonexpansive mappings play an important role in nonlinear analysis and its applications. One useful method is the shrinking projection method introduced by Takahashi, Takeuchi, and Kubota [9]. Since then, many researchers have studied and generalized this method. In particular, Kimura and Takahashi [6] improved the method by using the concept of Mosco convergence and established strong convergence theorems for relatively nonexpansive mappings in Banach spaces.

However, the shrinking projection method requires the exact computation of metric projections at each iteration step, and this computation becomes increasingly difficult as the iteration proceeds. To solve this problem, Kimura [3] proposed the shrinking projection method with nonsummable errors, which allows errors inside the target sets; see also [4,5]. Inspired by Kimura [3], Takeuchi [10] introduced the shrinking projection method with allowable ranges, which also allows errors outside the target sets.

On the other hand, in the study of common fixed point approximation methods, Ibaraki and Takeuchi [2] extended the concept of quasi-nonexpansive mappings, which generalize nonexpansive mappings, to families of nonlinear mappings by using the notion of attractive points. Furthermore, Lin and Takahashi [7] extended the concept of attractive points from Hilbert spaces to Banach spaces.

Motivated by these works, we propose a new shrinking projection method that allows errors both inside and outside the target sets. Using this method, we establish strong convergence theorems for finding a common fixed point of a family of nonlinear mappings of quasi-nonexpansive type in Banach spaces [1].

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(T.Ibaraki) Yokohama National University, Yokohama 240-8501, JAPAN
Email address: ibaraki@ynu.ac.jp

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