

GENERALIZE DANCŠ-HEGEDÜS-MEDVEGYEV'S PRINCIPLE WITH APPLICATIONS

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We establish an existence theorem for stationary points with a common fixed point for set-valued mappings by using sizing-up functions, under assumptions weaker than reflexivity and transitivity. Our result extends the Dancš–Hegedüs–Medvegyev principle [4] and related results of Lin–Du[12] and [16]. As applications, we obtain fixed point theorems of Subrahmanyam type [21], solvability results for variational relation and variational inclusion problems [2, 10, 14, 15, 17], a simultaneous quasi-Ekeland type variational principle [11], and existence results for Pareto-type optimization problems.

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