β-dimensional sharp maximal function and its applications

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Abstract

In this talk, I will introduce a β-dimensional sharp maximal operator defined using Hausdorff content:

$$\mathcal{M}_\beta^\# f(x) := \sup_{Q \ni x} \frac{1}{\ell(Q)^\beta} \inf_{c \in \mathbb{R}} \int_Q |f - c| \, d\mathcal{H}_\infty^\beta,$$

where the supremum is over cubes with sides parallel to the axes. I will present a Fefferman–Stein inequality in this setting, which is obtained via a new good-λ estimate adapted to the nonlinear structure of the Choquet integral. Time permitting, I will also discuss a pointwise comparison between this operator and the fractional maximal function applied to Riesz potentials, which leads to new insights into the local behavior of functions with bounded β-dimensional mean oscillation.