ON SCALARIZATION METHODS FOR SETS AND GENERALIZED CONE-CONTINUITY FOR SET-VALUED MAPS

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A composite function is a function which is the nesting of two or more functions to form a single new function. Such operation frequently preserves several mathematical properties of each nested function. For instance, a composition of continuous maps is continuous on topological spaces. From the view point of vector optimization and set optimization, this kind of inheritance by composite operations is important and useful to prove extended results and to get characterizations of optimal solutions through scalarization. This is a typical approach by which optimization problems with vector-valued or set-valued maps can be easily handled by converting vectors or sets into real numbers; see [2] and [3, 4].

Therefore, it is interesting to investigate the inheritance of cone continuity for set-valued maps via general scalarization functions for sets shown in [1].

On the other hand, by transforming set relations to twofold scalar optimization problems of some (real-valued) characterizing functions, the set-relations in [7] can be established by solving real-valued twofold optimization problems; see [5]. For each of the six set-relations, some necessary conditions and sufficient conditions are established on the negativity or the nonpositivity of the associated twofold scalar problems.

The aim of this talk based on [1, 5] is to intrduce the mechanism by which composite functions of a set-valued map and a scalarization function transmit semicontinuity of parent set-valued maps through several scalarization functions for sets, and to show the method how to characterize the set-relation through some characterizing functions for the convex cone.

References

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