DIRECTIONAL DERIVATIVE FOR SET-VALUED MAPS WITH WEIGHTED SET ORDER RELATIONS

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Directional derivatives of a set-valued map are very useful for deriving first-order necessary and sufficient conditions for set optimization problems. In this talk, we propose a new directional derivative for set-valued maps based on the weighted set order relations. We define a Hausdorff-type distance by means of nonlinear scalarization functions with respect to weighted set order relations and present some of its properties. By utilizing this Hausdorff-type distance, we propose a novel approach to computing the directional derivative of set-valued maps with weighted set order relations. We derive some nice properties from this directional derivative and use them to obtain the necessary and sufficient optimality conditions for the set optimization problem. As we know, the weighted set order relation is a mixture of lower and upper set order relations, the new directional derivative can be treated as a weighting directional derivative to balance out a possible gap in studying set optimization problems with lower and upper set order relations.

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