FIXED POINT THEOREM FOR WEIGHTED GENERALIZED PSEUDOCONTRACTIONS IN METRIC SPACES

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Let (X, d) be a metric space. A mapping T from X into itself is said to be widely more generalized hybrid if there exist real numbers $\alpha, \beta, \gamma, \delta, \varepsilon$ and ζ such that

$$\begin{aligned} &\alpha d(Tx,Ty)^2 + \beta d(x,Ty)^2 + \gamma d(Tx,y)^2 + \delta d(x,y)^2 \\ &\qquad +\varepsilon d(x,Tx)^2 + \zeta d(y,Ty)^2 \\ &\le 0 \end{aligned}$$

for any $x, y \in X$. Such a mapping is called an $(\alpha, \beta, \gamma, \delta, \varepsilon, \zeta)$-widely more generalized hybrid mapping.

Furthermore, we proved

Theorem 1. *Let X be a complete metric space and let T be an $(\alpha, \beta, \gamma, \delta, \varepsilon, \zeta)$-widely more generalized hybrid mapping from X into itself. Suppose that therre exists $\lambda \in [0, 1]$ such that*

(1) $\alpha + (1-\lambda)\varepsilon + \lambda\zeta + 2\min\{\lambda\beta + (1-\lambda)\gamma, 0\} \ge 0$;

(2) $\alpha + \delta + \varepsilon + \zeta + 4\min\{\lambda\beta + (1-\lambda)\gamma, 0\} > 0$;

(3) $\alpha + (1-\lambda)(\beta + \zeta) + \lambda(\gamma + \varepsilon) > 0$.

Then T has a fixed point u, where $u = \lim_{n\to\infty} T^n x$ for any $x \in X$. Additionally, if $\alpha + \beta + \gamma + \delta > 0$, then T has a unique fixed point.

In this talk, we introduce weighted generalized pseudocontractions and show a fixxed point theorem.

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