Set-Valued Fan-Takahashi Inequalities with Set-Relations Based on Scalarization Methods

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In convex analysis and optimization theory, Fan-Takahashi minimax inequality plays a key role to solve equilibrium problems and other problems. Let X be a nonempty compact convex subset of a Hausdorff topological vector space and $f\colon X \times X \to \mathbb{R}$. Fan-Takahashi minimax inequality is: if f satisfies the following conditions:

(a) for each fixed $y \in X$, $f(\cdot, y)$ is lower semicontinuous,
(b) for each fixed $x \in X$, $f(x, \cdot)$ is quasi concave,
(c) $f(x, x) \leq 0$ for all $x \in X$,

then there exists $\bar{x} \in X$ such that $f(\bar{x}, y) \leq 0$ for all $y \in X$.

About a quarter of a century ago, Georgiev and Tanaka [2] extended the minimax inequality to the form of set-valued maps. After that, Kuwano, Tanaka, and Yamada [4] constructed the result of four types of set-valued minimax inequalities with set relations [3], which are binary relations depending on a given convex cone. However, this result is limited to the case of specific scalarization functions. To obtain more practical results, we need to replace them into more general scalarization functions. In addition, Dechboon and Tanaka [1] proposed generalized semi-continuity to inherit properties of cone continuity for set-valued maps.

The aim of this talk is to explain Fan-Takahashi minimax inequality and our results in detail.

References

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