Recent results in vector optimization: applications and methods

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Abstract

Vector optimization involves minimizing or maximizing multiple objective functions under a preference order induced by a closed, convex, and pointed cone. When this cone is the nonnegative orthant, the problem is referred to as multiobjective optimization. In such cases, the goal is to find Pareto or weakly Pareto optimal solutions, which represent trade-offs among the objectives. Among the methods for solving vector or multiobjective optimization problems with theoretical guarantees, we can cite the scalarization techniques, such as the weighting and the "-constraint methods, as well as the descent-type methods, which are essentially extensions of scalar-valued algorithms to vector-valued settings. In this talk, we present two approaches involving different classes of vector optimization problems.

The first problem we consider is the vector optimization with continuous real variables and easily projectable constraints. We present the projected gradient method, which is wellsuited suitable for such problems, focusing on the subproblems that must be solved at each iteration. We then discuss the case of matrix-valued objective functions and demonstrate how the method can be applied to problems arising in experiment design applications. In the second part of the presentation, we shift our focus to multiobjective optimization with discrete structure, where many properties valid in the continuous setting no longer hold. We then examine a simple case using the concept of M-convexity, with an application in matching for bipartite graphs. In this case, we show that the entire Pareto set can be explicitly obtained.