Conjugate duality in set optimization via nonlinear scalarization

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Set optimization problems, a category of optimization problems, are a natural extension of multi-objective optimization problems and interval analysis. It is an emerging field that has been in the research field for over 20 years. The "set order relation," which is a natural extension of vector ordering, is a kind of preference in social choice theory. In 2014, Ide et al.[4] indicated that multi-objective robust optimization problems can be converted into applications of set optimization problems. Also in 2014, Hamel et al.[3] discovered that a special class of set optimization problems has a lattice structure. The nonlinear scalarization function (utility function) of vectors proposed by Gerstewitz (Tammer)[5] has the advantage of being able to fully characterize Pareto optimal solutions.

In this talk, we present the following topics[1]:

(a) We introduce nonlinear scalarizing techniques for sets as generalizations of Gerstewitz's scalarizing function for the vector-valued case. A detailed historical perspective and analysis of various characterizations of set order relations using nonlinear scalarization are given.

(b) We propose two new notions of set-valued conjugate maps based on lower (l-type) and upper (u-type) set order relations in a Hilbert space setting. This leads to four notions of set-valued bi-conjugate maps and corresponding weak duality theorems.

(c) Using nonlinear scalarizing techniques for sets, we propose strong duality theorems.

In addition, we will introduce the author's latest research results[2].

References

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