The 11th East Asia Number Theory

Geometric Gauss sums and Gross-Koblitz formula over function fields

Ting-Wei Chang (National Tsing Hua University)

Abstract:
Let p be an odd prime number. Classically, the Gross-Koblitz formula expresses the product of specific p-adic gamma values in terms of Gauss sums. As a result, we obtain a class of algebraic relations among p-adic gamma values. On the function field side, Thakur introduced the so-called "arithmetic" Gauss sums and derived a Gross-Koblitz-type formula for v-adic arithmetic gamma values. In this talk, we introduce the "geometric" Gauss sums over function fields, and present a geometric version of Gross-Koblitz formula. The primary key to our proof will also be discussed if time permits.

Arakelov geometry over adelic curves

Huayi Chen (Westlake University)

Abstract:
Arakelov geometry is a branch of arithmetic geometry where one combines different geometric methods to study varieties over a global field, namely a number field or the field of rational functions over a regular projective curve. In this talk, I will explain a joint work with Atsushi Moriwaki, where we develop an Arakelov geometry over any countable field equipped with an adelic structure. Then I will focus on some results of the geometry of numbers of adelic curves.

On the Blasius conjecture for the twisted standard L-functions

Shih-Yu Chen (National Tsing Hua University)

Abstract:
The behavior of periods of motives under twisting by Artin motives was studied in depth by Blasius. Building on Deligne's conjecture concerning the critical values of motivic L-functions, Blasius proposed a refined conjecture on the algebraicity of the critical values of twisted motivic L-functions, as well as its automorphic analogue in the setting of standard L-functions of algebraic cuspidal automorphic representations twisted by Artin representations. In this talk, we present results establishing new cases of the automorphic analogue under suitable regularity assumptions.

On Sidon sets

Jinhui Fang (Nanjing Normal University)

Abstract:
A Sidon set is a set A of positive integers with the property that all the sums $a + a'$ with $a, a' \in A$ and $a \leq a'$ are distinct. In this talk, we will present our recent results on Sidon sets.

Vanishing of Brauer groups of moduli stacks of stable curves

Kazuhiro Ito (Tohoku University)

Abstract:
I will discuss certain vanishing results for Brauer groups of smooth proper Deligne-Mumford stacks over the ring of integers $\mathbb{Z}$. As a main result, I will prove that the Brauer group of the moduli stack of stable curves of genus three is trivial. This complements previous results for other genera, for which the vanishing is (essentially) already known. The proof reduces to constructing a smooth plane quartic curve over a non-archimedean local field of equal characteristic with some explicit conditions, and to computing certain Hilbert symbols associated with it. This is joint work with Sebastian Bartling (University of Duisburg–Essen).

The average analytic rank of elliptic curves with a prescribed level structure

Keunyoung Jeong (Chonnam National University)

Abstract:
Heuristically, the average rank of elliptic curves over a number field is expected to be 1/2. The same principle applies to families of elliptic curves with prescribed level structures. In this talk, we present an explicit upper bound on the average rank of elliptic curves over a number field with a prescribed torsion subgroup, assuming the Hasse–Weil conjecture and the Riemann hypothesis for elliptic curve L-functions. We restrict to congruence subgroups Γ for which the associated moduli stack $\mathcal{X}_\Gamma$ is isomorphic to the projective line, thereby reducing the problem to counting rational points on a weighted projective space subject to local conditions. This is joint work with Peter J. Cho and Junyeong Park.

Sums of generalized polygonal numbers of almost prime length

Daejun Kim (Korea University)

Abstract:
Extending Lagrange's four-square theorem, it is expected that every sufficiently large integer congruent to 4 modulo 24 can be written as a sum of four squares of prime numbers. It has been shown that such integers can be expressed as a sum of four squares of integers, each with fewer than five prime factors. In this talk, we discuss an analogous problem concerning sums of three generalized m-gonal numbers, where parameters are restricted to integers with a bounded number of prime divisors. With some restriction modulo 30, we show that a density one set of integers can be represented as such a sum, where the parameters are restricted to have at most 6361 prime factors. This is joint work with Soumyarup Banerjee and Ben Kane.

Harder's conjecture and liftings of Siegel modular forms

Chul-hee Lee (Korea Institute for Advanced Study)

Abstract:
Harder's conjecture predicts surprising congruences between the Hecke eigenvalues of a classical modular form and a vector-valued Siegel modular forms of degree 2. We will discuss approaches to prove the conjecture in some cases using liftings of Siegel modular forms such as the Ikeda and Miyawaki lifts within the framework of Arthur's endoscopic classification of automorphic representations. This is joint work with H. Katsurada.

Cohen–Lenstra heuristics and random p-adic matrices with fixed zero entries

Jungin Lee (Ajou University)

Abstract:
Motivated by the Cohen–Lenstra heuristics, Friedman and Washington studied the distribution of the cokernels of random matrices over the ring of p-adic integers. This has been generalized in many directions, as well as some applications to the distribution of random algebraic objects.

In this talk, first we give an overview of random matrix theory over the ring of p-adic integers. After that, we investigate the distribution of the cokernels of random p-adic matrices with fixed zero entries. We provide several results on the minimal number of random entries required for the Cohen–Lenstra distribution. A part of this talk is based on the joint work with Dong Yeap Kang and Myungjun Yu.

The number of Maass forms with exceptional eigenvalues

Youngmin Lee (Korea Institute for Advanced Study)

Abstract:
The Ramanujan-Petersson conjecture predicts that cuspidal automorphic representations are tempered at all places. In the case of Maass forms, the conjecture at the archimedean place

is equivalent to the condition that the Laplace eigenvalue of a Maass form is greater than or equal to $\frac{1}{4}$, which is known as Selberg's eigenvalue conjecture. In this talk, we present results on upper bounds for the number of Maass forms with exceptional eigenvalues less than $\frac{1}{4}$. Furthermore, by interpreting Maass forms in terms of automorphic representations of GL_2, we describe an extension of these results to general number fields. These results are obtained using the Arthur-Selberg trace formula. This is joint work with Dohoon Choi, Min Lee, and Subong Lim.

Involution on a quotient space of multiple zeta values in positive characteristic

Yoshinori Mishiba (Tohoku University)

Abstract:
Multiple zeta values (MZVs) in positive characteristic were introduced by Thakur around 2004 as function field analogues of real-valued MZVs. In this talk, I will introduce multiple zeta dagger values, which can be regarded as "dual objects" to MZVs in a certain sense. Using these values, I will construct an involution on the quotient space of all MZVs in positive characteristic modulo the ideal generated by the single zeta value at $q-1$, where q is the order of the constant field of the function field. I will also explain that such a phenomenon may not occur in the characteristic zero case.

On a homotopic perspective on ramification and irregularity

Hiroyasu Miyazaki (NTT Institute for Fundamental Mathematics)

Abstract:
The $\mathbb{A}^1$-homotopy thoery (also called the motivic homotopy theory), which was initiated by Morel-Voevodsky, is an algebro-geometric analogue of the homotopy theory of topological spaces. The fundamental idea of this theory is to replace topological spaces by smooth algebraic varieties, and the notion of homotopy by the $\mathbb{A}^1$-homotopy, which uses the affine line $\mathbb{A}^1$ as the parameter space instead of the closed interval. The $\mathbb{A}^1$-homotopy thoery is a very powerful and universal framework to study cohomological invariants in algebraic geometry. However, there is a defect that it captures only $\mathbb{A}^1$-homotopy-invariant phenomena, and we lose some important information, e.g., wild ramification and irregular singularity. In this talk, I will propose a generalization of the $\mathbb{A}^1$-homotopy theory that captures non-$\mathbb{A}^1$-homotopy invariant phenomena. In particular, I will explain a universal construction which recovers a ramification filtration on the Pontryagin dual of the abelian étale fundamental group, and an irregularity filtration on the sheaf of rank 1 connections at the same time. This talk is based on a recent joint work with Junnosuke Koizumi and Shuji Saito.

Overgroups of the arboreal representation of PCF polynomials

Wayne Peng (National Central University)

Abstract:
Consider a number field K and a rational function f of degree greater than 1 over K. By taking preimages of $\alpha \in K$ under successive iterates of f, an infinite d-ary tree T_∞ rooted at α can be constructed. An edge is assigned between two preimages x and y if $f(x) = y$. The absolute Galois group of K, acting on T_∞ through tree automorphisms, generates a subgroup $\mathrm{Gal}_f^\infty(\alpha)$ in the group of all automorphisms of T_∞, $\mathrm{Aut}(T_\infty)$.

We have discovered a new class of natural overgroups in which the image of the Galois representation attached to a PCF polynomial must reside. Moreover, we have found that the image of the Galois representation of a new PCF polynomial is isomorphic to one of these overgroups. We also investigate the structure of these overgroups for specific maps, such as normalized dynamical Belyi polynomials, and show that the normal subgroups of these overgroups form a unique chief series. This allows us to bound the number of generators

through group-theoretic analysis.

Bounded generation: a Diophantine approximation approach

Jinbo Ren (Xiaman University)

Abstract:
An abstract group is said to have the *bounded generation* property (BG) if it can be written as a product of finitely many cyclic subgroups. Being a purely combinatorial notion, bounded generation has close relation with many group theoretical problems including semi-simple rigidity, Kazhdan's property (T) and Serre's congruence subgroup problem. This talk is devoted to explaining how to use the Laurent's theorem in Diophantine approximation to prove that an infinite S-arithmetic subgroup of an anisotropic linear algebraic group G over a number field K *never* has (BG). Moreover, I will introduce our newly obtained asymptotic formula for counting the elements of a "purely exponential parametrization" (PEP) set inside $GL_n(K)$ (K is a number field) when ordered by heights, together with some applications of this formula. The novelty of this project relies on the deep subspace theorem by Schlickewei-Schmidt as well as the theory of generic elements by Prasad-Rapinchuk. This is joint work with Corvaja, Demeio, Rapinchuk and Zannier.

Families preserving isomorphisms via techniques in anabelian geometry

Koichiro Sawada (RIMS, Kyoto University)

Abstract:
For an isomorphism between closed subgroups of a profinite group G, if the image of any pro-cyclic subgroup I via this isomorphism is conjugate to I in G, then we shall say that this isomorphism is "families preserving" in G.

Jarden and Ritter showed that, for a certain type of profinite group G—that includes the absolute Galois group of a p-adic local field and the étale fundamental group of hyperbolic curves—, every normal automorphism of G is inner. Their proof was divided into two steps: first showing that every normal automorphism is "families preserving" (in G), and then showing that every "families preserving" automorphism is inner.

In this talk, we discuss, from an anabelian geometrical point of view, whether a "families preserving" isomorphism between closed subgroups of a profinite group, such as the absolute Galois group of a certain field (Hilbertian field, Henselian discrete valuation field of positive residue characteristic) or its quotient, is induced from an inner automorphism. This is a joint work with Arata Minamide and Shota Tsujimura.

Some power series identities obtained from affine Lie superalgebras

Miyu Suzuki (Kyoto University)

Abstract:
The denominator identity is a combinatorial identity of power series attached to the root system of a Lie algebra. Macdonald identities, which include many famous q-series identities such as Jacobi triple product identity, are known to be derived from the denominator identities for affine Lie algebras. In 1994, Kac and Wakimoto formulated the denominator identity for affine Lie superalgebras and studied related q-series identities. However, their calculation was not completed. In this talk, we treat the remaining cases of affine Lie superalgebras. We also provide a different proof for the same q-series identities, based on the recently developed technique of indefinite theta functions.

TBA

Zhiyu Tian (Peking University)

Abstract:

Intersections of Hecke correspondences on the modular varieties of $\mathcal{D}$-elliptic sheaves

Ozge Ulkem (Academia Sinica)

Abstract:
In the classical setting, the well-known Hurwitz Kronecker class number relation expresses the 'finite part' of the intersection of cycles on the modular curve of full level in terms of Hurwitz class number. In the function field setting the Hurwitz Kronecker class number relation for Drinfeld modular curves is worked out by Wang and Yu. In this talk we will explore this phenomenon for a generalization of Drinfeld modular varieties, the modular varieties of '$\mathcal{D}$-elliptic sheaves'. This is a joint work with Fu-Tsun Wei.

Class number relations and mock modular forms

Yifan Yang (National Taiwan University)

Abstract:
In the talk we will use properties of mock modular forms to prove class number relations that are higher-dimensional analogues of the classical Hurwitz-Kronecker class number relations. This is a joint work with Bringmann, Guo, Kane, and Mertens.