Analysis of Vector Equilibrium Problems with Partial Orders Induced by Certain Classes of Cones

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Abstract

This thesis establishes a comprehensive analytical framework for vector equilibrium problems (VEPs) governed by cone-induced partial orders, with particular emphasis on polyhedral cones and p-order cones. The research systematically develops three interconnected directions.

First, new regularized gap functions are constructed, and sharp error bounds are established for VEPs associated with p-order cones on Hadamard manifolds. By explicitly exploiting the nonlinear geometry of the ℓ_p-norm cones, the classical error bound theory for scalar equilibrium problems and variational inequalities is successfully extended to vector settings ordered by these non-polyhedral cones.

Second, continuous-time dynamical approaches are developed for solving VEPs ordered by polyhedral cones, including ordinary differential systems and fractional-order neurodynamic models involving Caputo derivatives. The global convergence of trajectories to the solution sets is rigorously proved. In the fractional framework, Mittag–Leffler stability is established, highlighting the intrinsic advantages of memory-dependent dynamics.

Third, directional Levitin–Polyak well-posedness is generalized from operator-based variational inequalities to the broader bifunction formulation of VEPs. Using directional minimal time functions and cone-geometric analysis, this extension clarifies directional stability and robustness under matrix-induced partial orderings, thereby connecting directional convergence with residual gap function estimates.

Overall, these results deepen the theoretical foundations of cone-ordered equilibrium theory and provide stable analytical tools for the investigation of complex optimization and network equilibrium models.