Mini Optimization Workshop

Department of Mathematics
National Taiwan Normal University

June 3, 2025

Sponsored by

College of Science, National Taiwan Normal University
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Organized by

Jein-Shan Chen

Table 1: Schedule on June 3, 2025. Place: M212, Mathematics Department Building

	Speaker	**Title**	**Chair**
14:00 | 14:30	Narin Petrot	Set Optimization Perspectives in Bilevel Problems: Existence and Approximation Concepts	Yu-Lin Chang
14:30 | 15:00	Nimit Nimana	A Simultaneous Quasi-Subgradient Method for Minimizing Convex Functions with Quasi-Convex Functional Constraints	Yu-Lin Chang
15:00 | 15:30	*Tea Break*		
15:30 | 16:00	Wei-Shih Du	On multi-comparatively quasi-contractions with applications	Yu-Lin Chang
16:00 | 16:30	Tipsuda Arunrat	An Iterative Regularized Subgradient Algorithm with Information Delay for Solving Bilevel Convex Optimization Problems	Yu-Lin Chang
16:30 | 17:00	*Free Discussion*		

Set Optimization Perspectives in Bilevel Problems: Existence and Approximation Concepts

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Abstract. Bilevel optimization frequently entails a set-valued lower-level solution map, creating analytical and computational hurdles. In this presentation, we adopt a set optimization framework to establish and examine two main solution concepts: optimistic solutions (l-type) and robust solutions (u-type). We prove existence theorems for both, emphasizing the challenges that arise when the lower-level problem (e.g. a linear program with box constraints) leads to discontinuous solution maps or reduces to a trivial constant map—conditions that often impede the existence of robust solutions. To address cases where robust solutions are unattainable, we introduce an *approximate* solution concept and demonstrate its existence, thus providing a more flexible and realistic approach to complex bilevel structures.

Key words: Vector-valued bilevel optimization; set optimization; optimistic bilevel solution; robust bilevel solution; robust bilevel approximate solution

References:

[1] Kuroiwa, D., Petrot, N., Seto, K.; Solution Concepts for Bilevel Optimization Problems via Frameworks of Set Optimization. (Submitted).

A Simultaneous Quasi-Subgradient Method for Minimizing Convex Functions with Quasi-Convex Functional Constraints

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Abstract. In this work, we consider a convex minimization problem over the intersection of a compact convex simple set and a finite intersection of sublevel sets of quasi-convex functions. We propose the quasi-subgradient type method which separately deals with the objective function and a simple constrained set through a subgradient projection scheme and then performs parallel feasibility updates of constrained functions via a quasi-subgradient scheme with the appropriate weight function. This strategy provides a straightforward computation since we need not solve a subproblem to determine the metric projection onto the whole constrained set. Focusing on the convergence results, we prove subsequence convergence to the optimal solution of the considered problem and also establish the convergence rate for functional value to the optimal value. Additionally, by imposing the Hölder error bound property, we prove the convergence of the whole sequences to the optimal solution. We finally perform a numerical example to demonstrate the convergence behaviors of the proposed method for various choices of relating parameters and weight functions.

Key words: Nonsmooth optimization, Quasi-convex function, Quasi-convex subdifferential, Simultaneous method, Subgradient method, Functional constraints

On multi-comparatively quasi-contractions with applications

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Abstract. In this talk, we will introduce a new class of quasi-contractions, called multi-comparatively quasi-contractions, which includes many known multivalued contractions as special cases. We establish new sufficient conditions for multi-comparatively quasi-contractions and study its applications in fixed point theory.

Key words: simultaneous generalization, w-distance, τ-function, e-distance, e^0-metric, $\mathcal{MT}(\lambda)$-function, multi-comparatively quasi-contraction, p-approximate fixed point property, Banach contraction principle, Berinde-Berinde's fixed point theorem, Mizoguchi-Takahashi's fixed point theorem, Nadler's fixed point theorem, Kannan's fixed point theorem, Chatterjea's fixed point theorem.

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An Iterative Regularized Subgradient Algorithm with Information Delay for Solving Bilevel Convex Optimization Problems

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Abstract. This work focuses on the bilevel convex optimization problem. We introduce an iterative regularized subgradient algorithm to address this issue, allowing for the delayed utilization of subgradients for inner-level objective functions. With appropriate assumptions, we prove the convergence of the sequence generated by this method to the problem's solution and provide its convergence rate. Additionally, we leverage a similar incremental technique to propose a method for solving the bilevel convex optimization problem, particularly when the inner-level objective function comprises a finite sum of convex functions. Finally, we present numerical experiments on the image inpainting problem to illustrate the practical applications of our method. The results demonstrate that, despite requiring less time per iteration, the delayed subgradient method achieves comparable convergence to its non-delayed counterpart when delays are appropriately chosen.

Key words: Delay, Bilevel optimization, Subgradient method