Set Optimization Perspectives in Bilevel Problems: Existence and Approximation Concepts

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Abstract

Bilevel optimization frequently entails a set-valued lower-level solution map, creating analytical and computational hurdles. In this presentation, we adopt a set optimization framework to establish and examine two main solution concepts: optimistic solutions (l-type) and robust solutions (u-type). We prove existence theorems for both, emphasizing the challenges that arise when the lower-level problem (e.g.\ a linear program with box constraints) leads to discontinuous solution maps or reduces to a trivial constant map—conditions that often impede the existence of robust solutions. To address cases where robust solutions are unattainable, we introduce an \textit{approximate} solution concept and demonstrate its existence, thus providing a more flexible and realistic approach to complex bilevel structures.

Keywords: Vector-valued bilevel optimization; set optimization; optimistic bilevel solution; robust bilevel solution; robust bilevel approximate solution

References:

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