Generalized convexity for spectral functions on Euclidean Jordan algebras

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Abstract

In a Euclidean Jordan algebra $\mathcal{V}$, a spectral function $G : \mathcal{V} \to \mathbb{R}$ is defined as a function that depends solely on the eigenvalues of its argument. Formally, such a function takes the form $G = f \circ \lambda$, where $f : \mathbb{R}^n \to \mathbb{R}$ is a (symmetric) function and $\lambda : \mathcal{V} \to \mathbb{R}^n$ denotes the eigenvalue mapping. It turns out that spectral functions are invariant under algebra automorphisms of $\mathcal{V}$. Due to their simple yet elegant structure and wide applicability, spectral functions play a crucial role not only in matrix theory but also in convex analysis, optimization, and beyond. It has been observed that G is (strictly) convex if and only if so is the associated function f, which we call a transfer principle for (strict) convexity. In this talk, we will explore analogous transfer principle for generalized notions of convexity, including strong convexity, quasi-convexity, pseudo-convexity, and related concepts.