Realistic Function Spaces for Physics

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Abstract: Modern mathematical physics models natural phenomena via the Calculus of Variations and Partial Differential Equations. The study of these energies and equations prompts the introduction of function spaces, where questions of interest are compactness, lower-semicontinuity, and estimates. While the original equations themselves are systems, for simplicity mathematicians first work with scalar analogues, where in certain regimes the natural compactness is insufficient to prove the desired estimates. The consideration of the failures of these estimates led to the introduction of function spaces where they are valid, which unfortunately are not physically motivated. In this talk I will discuss in more detail this history, and to introduce some function spaces which are physically motivated and also sufficiently robust to establish the desired properties. This talk is based on joint work with Dmitriy Stolyarov, "On dimension stable spaces of measures," https://arxiv.org/abs/2405.10728.

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