Mini Optimization Symposium 2024

Department of Mathematics
National Taiwan Normal University

December 10, 2024

Sponsored by

College of Science, National Taiwan Normal University
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Organized by

Jein-Shan Chen

Table 1: Schedule on December 10, 2024. Place: M212, Mathematics Department Building

	Speaker	Title
14:00 | 14:40	Pham Tien Son	Subdifferentials at infinity and applications in optimization
14:40 | 15:20	Juyoung Jeong	Commutation principles for nonsmooth variational problems on Euclidean Jordan algebras
15:20 | 15:40		*Tea Break*
15:40 | 16:20	Ruey-Lin Sheu	The Non-homogeneous Fisler-Calabi Theorem with Its Proof
16:20 | 17:00	Ching-Yu Yang	Novel constructions for closed convex cones through inequalities and support functions
17:00 | 17:20		*Free Discussion*

Subdifferentials at infinity and applications in optimization

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Abstract. In this work, the notions of *normal cones at infinity* to unbounded sets and *limiting and singular subdifferentials at infinity* for extended real value functions are introduced. Various calculus rules for these notions are established. A complete characterization of the Lipschitz continuity at infinity for lower semicontinuous functions is given. The obtained results are aimed ultimately at applications to diverse problems of optimization, such as optimality conditions, weak sharp minima, coercive properties and stability results.

Commutation principles for nonsmooth variational problems on Euclidean Jordan algebras

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Abstract. The commutation principle proved by Ramírez, Seeger, and Sossa in the setting of Euclidean Jordan algebras says that for a Fréchet differentiable function Θ and a spectral function F , any local minimizer or maximizer a of $\Theta + F$ over a spectral set ε operator commutes with the gradient of Θ at a. In this paper, we improve this commutation principle by allowing Θ to be nonsmooth with regularity assumptions over it. For example, for the case of local minimizer, we show that a operator commutes with some element of the limiting (Mordukhovich) subdifferential of Θ at a provided that Θ is subdifferentially regular at a satisfying a qualification condition. For the case of local maximizer, we prove that a operator commutes with each element of the (Fenchel) subdifferential of Θ at a whenever this subdifferential is nonempty. As an application, we characterize the local optimizers of shifted strictly convex spectral functions and norms over automorphism invariant sets.

The Non-homogeneous Fisler-Calabi Theorem with Its Proof

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Abstract. Given two n-variate quadratic functions $f(x) = x^T Ax + 2a^T x + a_0, g(x) = x^T Bx + 2b^T x + b_0$, we aim to characterize analytically whether or not

$$\{x \in \mathbb{R}^n : f(x) = 0\} \cap \{x \in \mathbb{R}^n : g(x) = 0\} = \emptyset.$$

When $n \geq 3$ and $a = b = 0, a_0 = b_0 = 0$, this is known as the famous Finsler-Calabi theorem[1,2] (1936-1964), in which case the Finsler-Calabi theorem asserts that $f = g = 0$ has no common solution other than the trivial one $x = 0$ if and only if there exists a positive definite matrix pencil $\alpha A + \beta B \succ 0$. In this talk, we generalize the result of the Finsler-Calabi theorem to non-homogeneous quadratic functions and also give the proof.

[1]P. Finsler, U¨ ber das vorkommen definiter und semidefiniter formen in scharen quadratischer formen, Commentarii Mathematici Helvetici, 9 (1936) pp. 188–192.

[2]E. Calabi, Linear systems of real quadratic forms, Proceedings of the American Mathematical Society, 15 (1964), pp. 844–846.

Novel constructions for closed convex cones through inequalities and support functions

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Abstract. Two novel ways to generate closed convex cones, the main ingredient of conic optimization, are proposed in this study. The first way is constructing closed convex cones via inequalities, whereas the second one is through support functions. The contribution of this article is twofold. One is opening up new ideas for looking into structures of closed convex cones. The other one is providing novel approaches and mediums for investigating conic optimization.