The Non-homogeneous Fisler-Calabi Theorem with Its Proof

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Given two n-variate quadratic functions $f(x) = x^TAx+2a^Tx+a_0,\ g(x) = x^TBx+2b^Tx+b_0$, we aim to characterize analytically whether or not

$$\{x \in R^n : f(x) = 0\} \cap \{x \in R^n : g(x) = 0\} = \emptyset.$$

When $n \geq 3$ and $a = b = 0,\ a_0 = b_0 = 0$, this is known as the famous Finsler-Calabi theorem[1,2] (1936-1964), in which case the Finsler-Calabi theorem asserts that $f = g = 0$ has no common solution other than the trivial one $x = 0$ if and only if there exists a positive definite matrix pencil $\alpha A+\beta B \succ 0$. In this talk, we generalize the result of the Finsler-Calabi theorem to non-homogeneous quadratic functions and also give the proof.

[1] P. Finsler, *Über das vorkommen definiter und semidefiniter formen in scharen quadratischer formen*, Commentarii Mathematici Helvetici, 9 (1936) pp. 188–192.

[2] E. Calabi, *Linear systems of real quadratic forms*, Proceedings of the American Mathematical Society, 15 (1964), pp. 844–846.