# Commutation principles for nonsmooth variational problems on Euclidean Jordan algebras

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## Abstract

The commutation principle proved by Ramírez, Seeger, and Sossa in the setting of Euclidean Jordan algebras says that for a Fréchet differentiable function $\Theta$ and a spectral function $F$, any local minimizer or maximizer $a$ of $\Theta + F$ over a spectral set $\mathcal{E}$ operator commutes with the gradient of $\Theta$ at $a$. In this paper, we improve this commutation principle by allowing $\Theta$ to be nonsmooth with regularity assumptions over it. For example, for the case of local minimizer, we show that $a$ operator commutes with some element of the limiting (Mordukhovich) subdifferential of $\Theta$ at $a$ provided that $\Theta$ is subdifferentially regular at $a$ satisfying a qualification condition. For the case of local maximizer, we prove that $a$ operator commutes with each element of the (Fenchel) subdifferential of $\Theta$ at $a$ whenever this subdifferential is nonempty. As an application, we characterize the local optimizers of shifted strictly convex spectral functions and norms over automorphism invariant sets.

# References

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