雙語數學誌

MATHLINGUAL

BILINGUAL MATHEMATICS MAGAZINE



Discover Vectors in Daily Life

Why Do We Miss Even When We Aim Carefully?
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Nov, 2026

A Message from the Dean

I am delighted to share with you the inaugural issue of MathLingual: Bilingual Mathematics Magazine.

As Dean of the College of Science at National Taiwan Normal University, I warmly welcome all of our teachers and students to this exciting new journey.

In science, mathematics is the fundamental language we use to understand how the world works. At the same time, English is the key that connects us to the global community. Bilingual education is therefore about much more than learning another language, it leads to develop new ways of thinking.

When we explore mathematics through a different language, we gain a fresh perspective on its logic, structure, and beauty. We begin to see that mathematical ideas transcend borders and that language can serve as a bridge to deeper understanding.

This magazine was created to provide a meaningful and inspiring learning opportunity. For teachers, it offers a new way to engage and motivate students. For students, it opens a window to a broader world of knowledge and discovery.

I hope that MathLingual will encourage you to look beyond the classroom, broaden your horizons, and experience the unique intersection where mathematics and language come together.

May you enjoy every issue and continue to explore the wonderful world of knowledge with curiosity, confidence, and an international vision.

Jein-Shan Chen

Dean, College of Science,
National Taiwan Normal University









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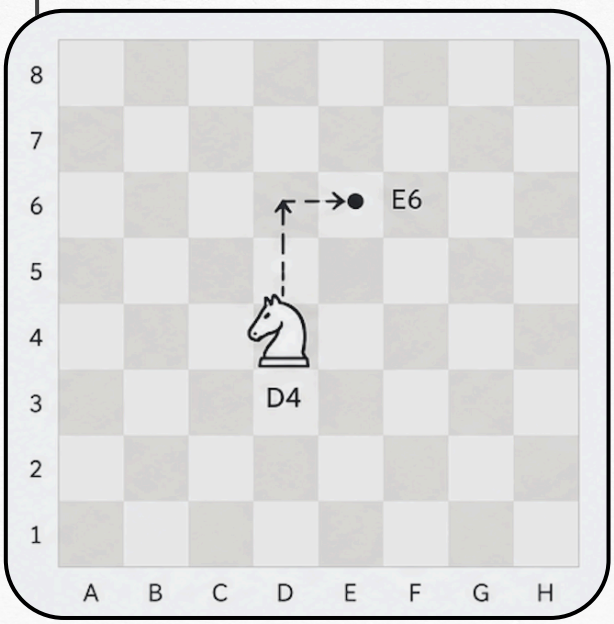
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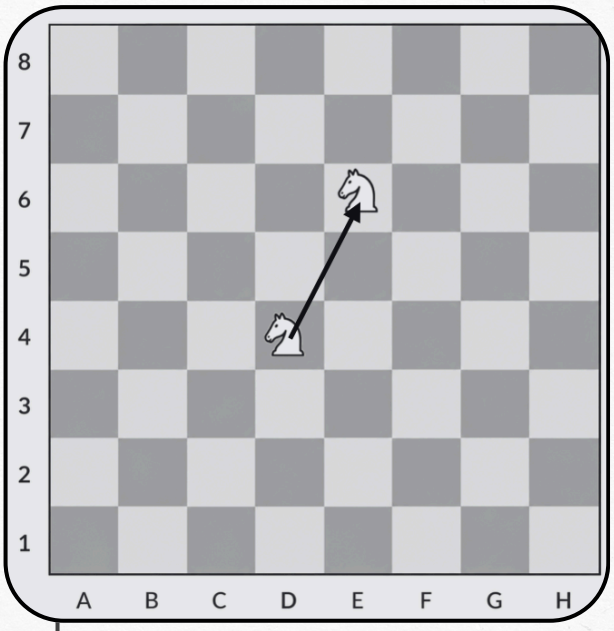
VECTORS

MAGNITUDE AND DIRECTION

A KNIGHT'S MOVE

In chess (西洋棋), a knight moves in an L-shape: two squares in one direction and one square perpendicular to it.





If we only care about the change in position, we can describe the move from D4 to E6 with a vector, represented by a single arrow.

Different representations preserve different information.

The L-shaped path tells us how the knight moves. The vector tells us both how much and which direction the postion changes.



Invisible Arrows

All Around Us



(01) Why Direction Matters?

In everyday life, we often use numbers to describe motion, distance, and force. However, a number alone does not always tell the whole story. When direction matters, we need a vector.

日常生活中的向量不一定真的畫成箭頭，但只要一項資訊同時包含「大小」與「方向」，我們就可以用向量描述。

(02) Speed or Velocity?

Speed（速率）means how fast an object is moving. It has magnitude, but no direction.

Velocity（速度）means both how fast an object is moving and in which direction.

Language Note

In everyday English, people sometimes use speed and velocity interchangeably.
In mathematics and physics, however, velocity always includes direction.



The car is moving at 60 kilometers per hour.

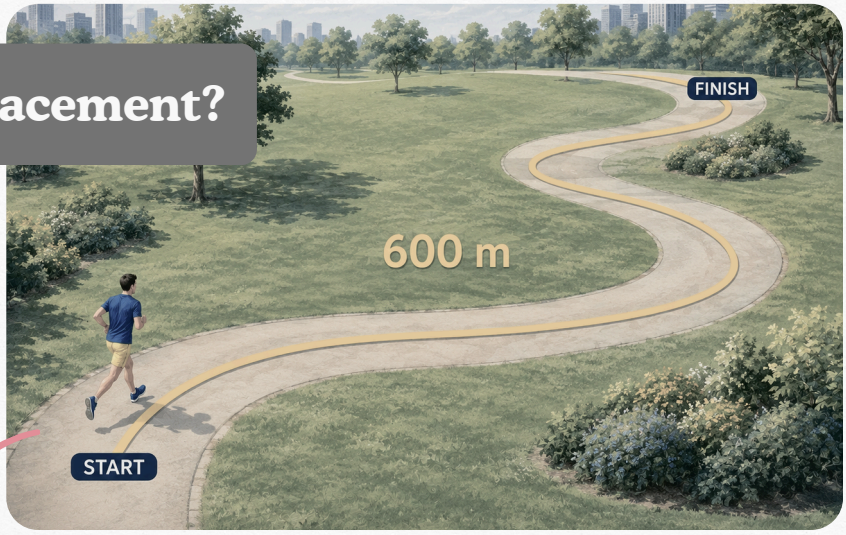


The car is moving EAST at 60 kilometers per hour.

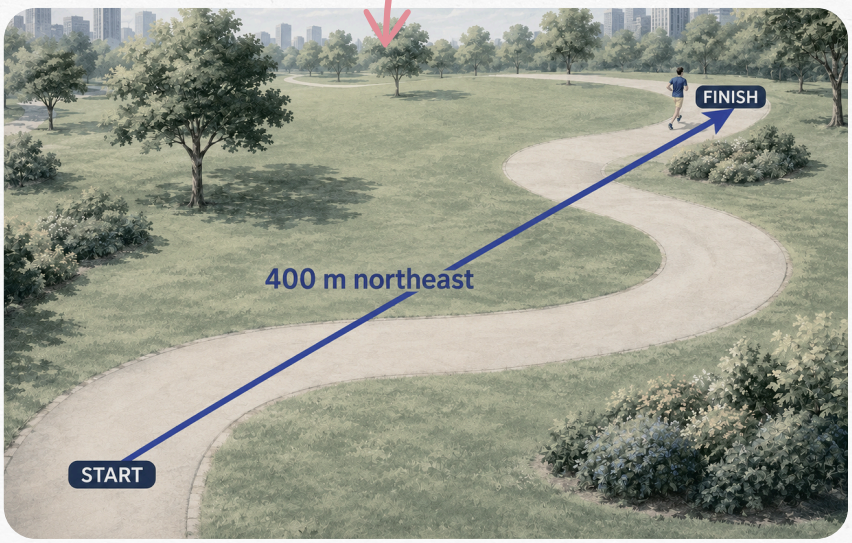
Invisible Arrows

All Around Us

(03) Distance or Displacement?



Distance measures the total length of the route.



Displacement describes the change in position.

(04) Did You Notice?

On a standard outdoor track, one full lap in Lane 1 is 400 meters. If a runner completes one full lap and returns to the starting point, the distance is 400 meters, but the displacement is zero.

Vector Language

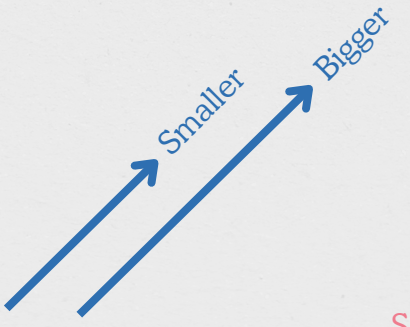
From Reality to Mathematics

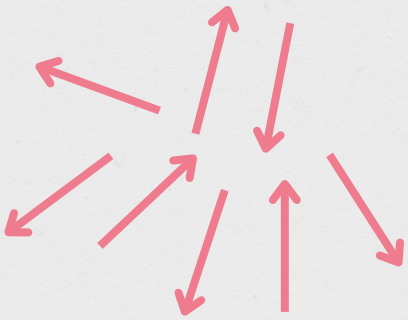
The previous pages showed motion, displacement, and force. Each example carries information about how much a quantity changes and which direction it goes. Mathematics gives us a compact way to express both ideas.

Abstract Definition

A vector describes a change from an initial point to a terminal point, with both *magnitude* and *direction*. Magnitude refers to the length of a vector, telling us how much or how far. Direction tells us which way the vector points.







Same size but various directions



Scalar or Vector?

Mass: 5 kg

Speed: 60 km/h

Temperature: 25°C

A: Magnitude only.

How to Name a Vector

Using Two points: A vector can be named using its starting point and ending point. For example, $\vec{AB}$ starts at point A and ends at point B. The order matters!

A is the initial point (起點), and B is the terminal point (終點). Reversing the order gives $\vec{BA}$, which has the same magnitude (大小) but points in the opposite direction.

Using a letter: Vector can also be represented by a lowercase bold letter, such as **v**, or a letter with an arrow above it, such as $\vec{v}$.



The zero vector (零向量) has a magnitude of 0 and is written as **0** or $\vec{o}$.
It has no specific direction.

Vector Language

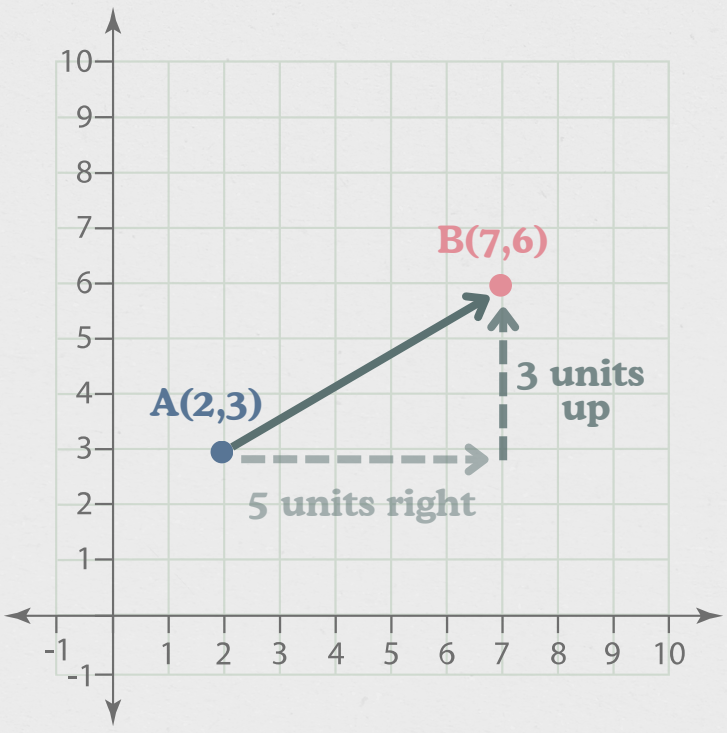
A vector can be represented visually by an arrow or numerically by its components. The first component describes the horizontal change, while the second describes the vertical change.

Component Form

When a vector begins at the origin $O(0, 0)$ and ends at $P(a, b)$, its component form is

$$\vec{OP} = (a, b).$$

First component: horizontal change.
Second component: vertical change.



From *A* to *B*

Terminal Coordinates
− Initial Coordinates
= Vector Components

$B(x_2, y_2) - A(x_1, y_1)$
$= \vec{AB}\ (x_2 - x_1, y_2 - y_1)$

Example:

Given two points $A(2, 3)$ and $B(7, 6)$, the components of

$\vec{AB} = (7-2, 6-3)$
$= (5, 3)$

Magnitude:

If $\mathbf{v} = (a, b)$, then

$$|\mathbf{v}| = \sqrt{a^2 + b^2}$$

For $\vec{AB} = (5, 3)$, its magnitude is

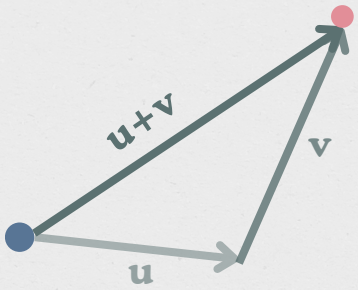
$$|\vec{AB}| = \sqrt{5^2 + 3^2} = \sqrt{43}$$



A vector's components describe its horizontal and vertical change.

Vector Language

How vectors combine, reverse, and scale



Vector operations describe several changes acting together. The diagrams show the geometry. Component formulas give the same result numerically.

Triangle Law

Place the initial point of **v** at the terminal point of **u**. The resultant vector connects the initial point of **u** to the terminal point of **v**

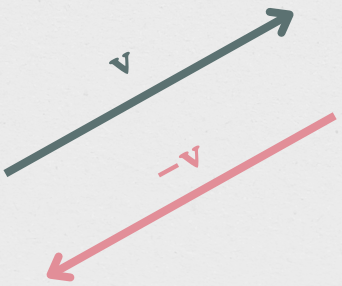
Component Rule

Given two vectors $\mathbf{u} = (a, b)$ and $\mathbf{v} = (c, d)$, their sum is found by adding the corresponding components:

$$\mathbf{u} + \mathbf{v} = (a + c, b + d)$$

Example:

$$(2, 1) + (1, 3) = (3, 4)$$



Same magnitude, opposite direction.

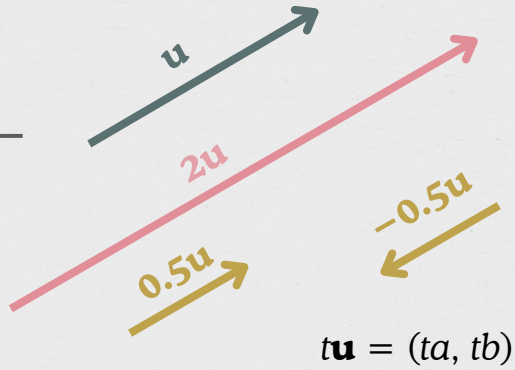
Subtraction and Opposite Vectors

Subtracting **v** means adding the opposite vector $-\mathbf{v}$.

$$\mathbf{u} - \mathbf{v} = \mathbf{u} + (-\mathbf{v})$$

Scalar Multiplication

Multiplying by a scalar changes the magnitude. A negative scalar also reverses the direction.



$t\mathbf{u} = (ta, tb)$

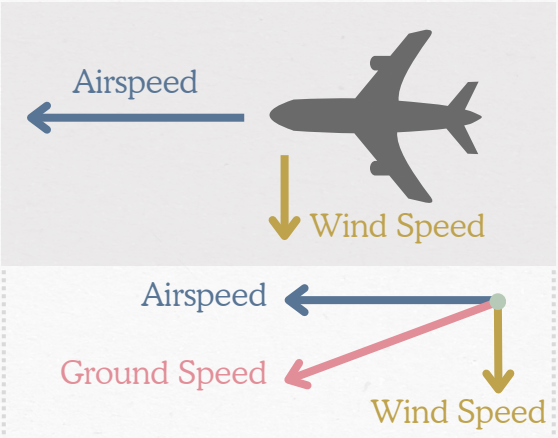


Add components to combine vectors. Multiply components to scale a vector.

Why Do Airplanes Drift Off Corse?

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Ground Speed =
Vector Sum of Airspeed and Wind Speed

Wind can change the path of an airplane. An airplane has its own airspeed, while the wind has its own speed and direction. Both can be represented as vectors.

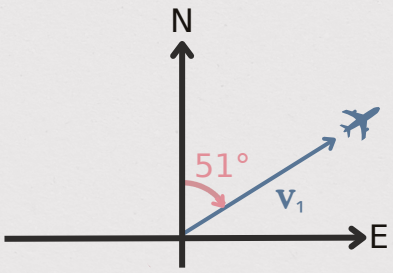
When a crosswind blows from the side, it pushes the airplane sideways, causing its actual path to differ from the direction it is facing. By using vector addition, we can find the airplane's actual ground speed and flight path.

That is why pilots need to pay attention to wind speed and direction and adjust the airplane's heading to stay on course. Math helps airplanes stay on the right path.

Think About It

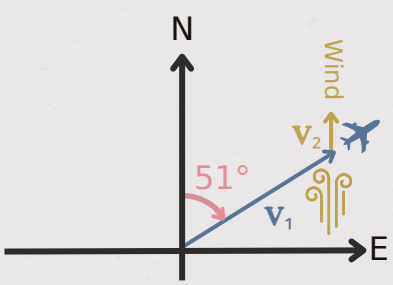


Imagine you are planning a flight from Taipei, Taiwan, to Tokyo, Japan.
The airplane needs to travel eastward across the United States. The pilot sets the airplane's speed at 600 miles per hour with a *bearing* of 51°
(Meaning 51° clockwise from north).



Adding Wind

At first, everything looks fine. But as the airplane begins its journey, the pilot receives a weather update:
"A strong wind is moving toward the north at 40 miles per hour."
The wind changes the airplane's actual motion over the ground.



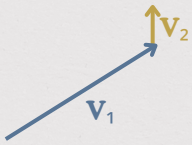
Find the velocity of the airplane.

$$\mathbf{V}_1 = 600\,(\cos 39°, \sin 39°)$$
$$\approx (466.26, 377.58)$$



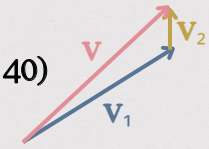
Find the velocity the wind.

$$\mathbf{V}_2 = 40\,(\cos 90°, \sin 90°)$$
$$\approx (0, 40)$$



Find the resultant velocity.

$$\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2$$
$$\approx (466.26, 377.58) + (0, 40)$$
$$= (466.26, 417.58)$$



The resultant speed of the airplane is

$$|\mathbf{V}| = \sqrt{466.26^2 + 417.58^2}$$
$$\approx 625.92$$

Find the direction angle θ.

$$\theta = \tan^{-1}\left(\frac{417.58}{466.26}\right)$$
$$\approx \tan^{-1}(0.8956)$$
$$\approx 41.8°$$

Therefore,

the resultant speed is about 625.92 miles per hour, and the true direction is 41.8°.

Vector addition is also used in:

- Sailing (wind and currents)
- Boats and ships (waves and currents)
- Sports (a golfer's shot affected by the wind, an archer's aim

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