

雙語教學主題(國中八年級下學期教材)平行線與截線所形成角的關係

Topic: Test questions for Angles formed by parallel lines and their transversals and the relationships among the angles

Vocabulary

CPCTC stands for <i>corresponding parts of the congruent triangles are congruent</i>		兩全等三角形的對應邊和對應角都對應相等	
transversal	截線	parallel	平行
intersect	相交	exterior angle	外角
interior angle	內角	vertical angle	對頂角
adjacent angle	鄰角	congruent	全等，相等
corresponding angle	同位角	alternate interior angle	內錯角
consecutive interior angle	同側內角	alternate exterior angle	外錯角
consecutive exterior angle	同側外角	the sum of interior angles theorem	內角和定理
respectively	按順序地	fundamental	基本的
arbitrary point	任意點	intersection point	交點
supplementary	互補	geometric	幾何的
common perpendicular line	公垂線	reflexive property	自身性，反身性
replacement	替換	lengthen	延長

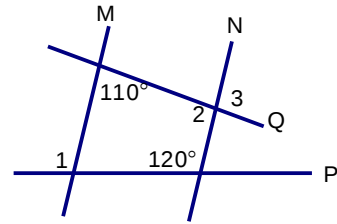
(我個人很喜歡這個 CPCTC 的表示法，否則要寫很多中文字...)

老師們好，這是平行截線相關的應用練習。這份教材的前半是題目加解答，後半是題目，方便老師們參考使用。老師們可以依據學生的需求選取適當的題目供學生做小測或是練習。

Q 1:

Given four lines M, N, P, and Q as shown in the figure.

$M \parallel N$, calculate the measure of $\angle 1$, $\angle 2$, and $\angle 3$.



ANSWER:

Sol:

$M \parallel N$, then

$$\angle 1 = 120^\circ \quad (\text{Corresponding angles are congruent.})$$

$$\angle 2 + 120^\circ = 180^\circ \quad (\text{Consecutive interior angles are supplementary.})$$

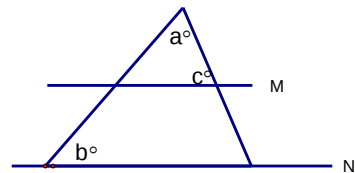
So $\angle 2 = 60^\circ$

$$\angle 3 = \angle 2 = 60^\circ \quad (\text{Vertical angles are congruent.})$$

Q 2:

Lines M and N are parallel. $a^\circ = 55^\circ$, $b^\circ = 45^\circ$.

Find the value of c



ANSWER:

Sol:

Note $\angle 1$ as shown in Figure 1.

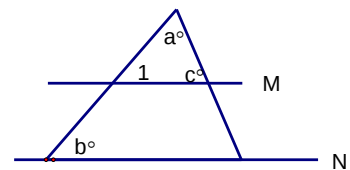


Figure 1

$M \parallel N$, then

$$\angle 1 = b^\circ = 45^\circ \quad (\text{Congruent corresponding angles})$$

And $\angle 1 + a^\circ + c^\circ = 180^\circ$ (The interior angle sum theorem.)

That is:

$$45^\circ + 55^\circ + c^\circ = 180^\circ$$

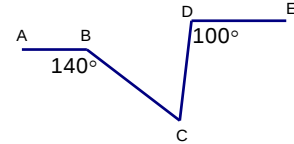
$$c^\circ = 80^\circ$$

$$c = 80^\circ$$

Q 3:

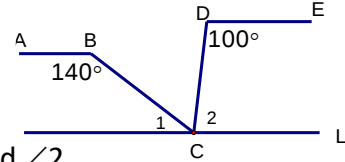
$\overline{AB} \parallel \overline{DE}$, $\angle B = 140^\circ$, $\angle D = 100^\circ$.

Find the measure of $\angle C$.



ANSWER:

Sol:



Construct line $L \parallel \overline{AB} \parallel \overline{DE}$ in Figure 1, and note $\angle 1$ and $\angle 2$.

$$\angle 1 + 140^\circ = 180^\circ \quad (\text{Consecutive interior angles are supplementary.})$$

$$\angle 2 + 100^\circ = 180^\circ \quad (\text{Consecutive interior angles are supplementary.})$$

Simplify the equations, we get

$$\angle 1 = 40^\circ$$

$$\angle 2 = 80^\circ$$

And $\angle 1 + \angle 2 + \angle BCD = 180^\circ$ (A straight angle is 180° .)

Then $40^\circ + 80^\circ + \angle BCD = 180^\circ$ (Replace $\angle 1$ with 40° and $\angle 2$ with 80°)

$$\angle BCD = 60^\circ \#$$

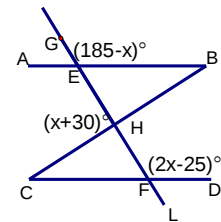
Q 4:

As shown in the figure, $\overline{AB} \parallel \overline{CD}$, line L intersects $\overline{AB}$ and $\overline{CD}$ at points E and F respectively. $\angle BEG = (185 - X)^\circ$, $\angle DFE = (2X - 25)^\circ$, and $\angle EHC = (X + 30)^\circ$

Find

(1) The value of x

(2) The measure of $\angle C$.



ANSWER:

Sol:

(1)

$\overline{AB} \parallel \overline{CD}$,

$$\angle BEG = \angle DFE \quad (\text{Congruent corresponding angles})$$

$$185 - x = 2x - 25 \quad (\text{Given})$$

Simplify

$$x = 70 \#$$

(2)

$$\angle EHC = \angle C + \angle CFH \cdots \textcircled{1} \quad (\text{Triangle exterior angle theorem.})$$

$$\angle EHC = (X + 30)^\circ = 100^\circ \quad (\text{Replace } x \text{ with } 70)$$

$$\angle CFH + \angle DFE = 180^\circ \quad (\text{Straight angles are } 180^\circ)$$

And $\angle DFE = (2x - 25)^\circ = 115^\circ$ (Replace x with 70)
 Then $\angle CFH + 115^\circ = 180^\circ$ (Replace $\angle DFE$ with 115°)
 $\angle CFH = 65^\circ$

From ① $\angle EHC = \angle C + \angle CFH$

Then

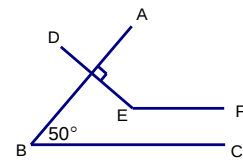
$$100^\circ = \angle C + 65^\circ \quad (\text{Replace } \angle EHC \text{ with } 100^\circ \text{ and } \angle CFH = 65^\circ)$$

We get

$$\angle C = 35^\circ \#$$

Q 5:

As shown in the figure, $\overline{AB} \perp \overline{DE}$, $\overline{BC} \parallel \overline{EF}$, and $\angle B = 50^\circ$.
 Find the measure of $\angle E$.



ANSWER:

Sol:

Construct $\overline{GE} \parallel \overline{BC}$ and intersects $\overline{AB}$ at point G
 as shown in Figure 1. Note $\angle 1 = \angle AGE$.

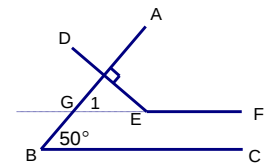


Figure 1

Then

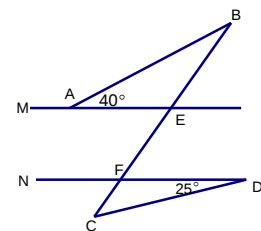
$$\angle 1 = \angle B = 50^\circ \quad (\text{Congruent corresponding angles})$$

$$\angle DEF = \angle 1 + 90^\circ \quad (\overline{AB} \perp \overline{DE} \text{ and exterior angle theorem})$$

$$\begin{aligned} \angle DEF &= 50^\circ + 90^\circ \\ &= 140^\circ \# \end{aligned}$$

Q 6:

Given parallel lines M and N, $\overline{BC}$ intersects lines M and N at points E and F respectively. If $\angle BAE = 40^\circ$, $\angle D = 25^\circ$.
 Calculate $\angle C - \angle B$.



ANSWER:

Sol:

$$\angle FEA = \angle B + \angle BAE \quad \dots \textcircled{1} \quad (\text{Exterior angle theorem})$$

$$\angle EFD = \angle C + \angle D \quad \dots \textcircled{2} \quad (\text{Exterior angle theorem})$$

$$\angle FEA = \angle EFD \quad (\text{Congruent alternate interior angles})$$

Then

$$\Rightarrow \angle B + \angle BAE = \angle FEA = \angle EFD = \angle C + \angle D \quad (\text{From } \textcircled{1}, \textcircled{2})$$

$$\angle B + 40^\circ = \angle C + 25^\circ \quad (\text{Replace } \angle BAE \text{ with } 40^\circ, \angle D \text{ with } 25^\circ)$$

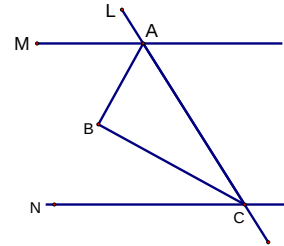
Therefore,

$$\begin{aligned} \angle C - \angle B &= 40^\circ - 25^\circ \\ &= 15^\circ \# \end{aligned}$$

Q 7:

Given lines M and N are parallel. Line L is a transversal and intersects lines M and N at points A and C respectively.

$\overline{AB}$ is the angle bisector of $\angle CAM$, and $\overline{BC}$ is the angle Bisector of $\angle ACN$. Find the measure of $\angle B$.



ANSWER:

Sol:

Note $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$ as shown in Figure 1.

$$\angle 1 = \angle 2 \quad (\overline{AB} \text{ is the angle bisector of } \angle CAM)$$

$$\angle 3 = \angle 4 \quad (\overline{BC} \text{ is the angle Bisector of } \angle ACN)$$

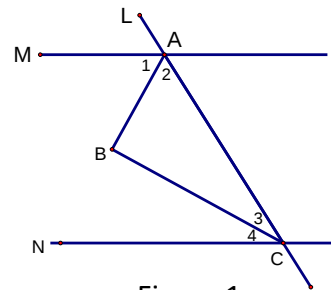


Figure 1

Since

$$\angle CAM + \angle ACN = 180^\circ \quad (\text{Consecutive interior angles are supplementary.})$$

$$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 180^\circ$$

That is

$$2\angle 2 + 2\angle 3 = 180^\circ \quad (\angle 1 = \angle 2, \angle 3 = \angle 4)$$

$$\angle 2 + \angle 3 = 90^\circ$$

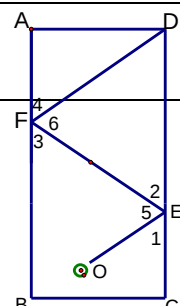
$$\text{And } \angle B + \angle 2 + \angle 3 = 180^\circ \quad (\text{Sum of the triangle interior angles theorem})$$

$$\text{So } \angle B = 90^\circ \#$$

We can see that the angle bisectors of two consecutive interior angles are perpendicular to each other.

Q 8:

As shown in the figure, a cue ball on a pool table is moving



along $\overrightarrow{OE}$ and hits the side $\overrightarrow{CD}$ at point E.

The cue ball rebounds along $\overrightarrow{EF}$ and hits the side

$\overrightarrow{AB}$ at point F. Then the cue ball rebounds again along $\overrightarrow{FD}$ and rolls into the pocket at point D.

If we know $\angle 1 = \angle 2$, $\angle 3 = \angle 4$, please explain whether

$\overrightarrow{OE}$ and $\overrightarrow{FD}$ are parallel.

ANSWER:

Sol:

$$\angle 1 + \angle 2 + \angle 5 = 180^\circ \quad (\overrightarrow{CD} \text{ is a segment.})$$

$$\angle 3 + \angle 4 + \angle 6 = 180^\circ \quad (\overrightarrow{AB} \text{ is a segment.})$$

Then

$$\angle 5 = 180^\circ - \angle 1 - \angle 2 = 180^\circ - \angle 3 - \angle 4 = \angle 6$$

$$\text{i.e. } \angle 5 = 180^\circ - 2\angle 2 = 180^\circ - 2\angle 3 = \angle 6 \quad (\text{Given } \angle 1 = \angle 2, \angle 3 = \angle 4)$$

$$\text{But } \angle 2 = \angle 3 \quad (\overrightarrow{AB} \parallel \overrightarrow{CD}, \text{ alternate interior angles are congruent.})$$

$$\Rightarrow \angle 5 = \angle 6$$

Therefore,

$\overrightarrow{OE}$ and $\overrightarrow{FD}$ are parallel

(Their alternate interior angles are congruent.)

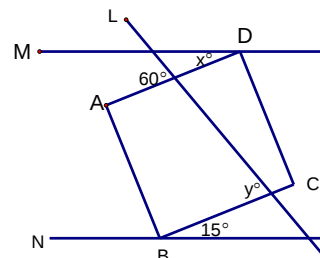
Q 9:

As shown in the figure, quadrilateral ABCD is a square.

Lines M and N are parallel. Points D and B lie on lines

M and N respectively.

Find the value of x and y.



ANSWER:

Sol:

Note $\angle 1$, and $\angle 2$ in Figure 1 as shown in Figure 1.

$$y^\circ = 60^\circ$$

$$\text{i.e. } y = 60$$

($\overrightarrow{AD} \parallel \overrightarrow{BC}$, congruent corresponding angles.) Figure 1

$$y^\circ = \angle 1 + 15^\circ \quad (\text{Triangle exterior angle theorem})$$

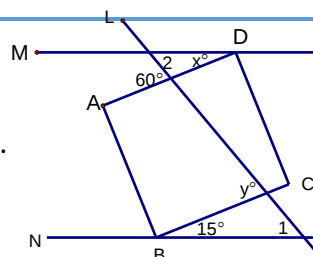
$$60^\circ = \angle 1 + 15^\circ \quad (\text{Replace } y \text{ with } 60)$$

$$\angle 1 = 45^\circ$$

$$\text{And } \angle 2 = \angle 1 = 45^\circ \quad (\text{Lines } M \parallel N, \text{ alternate interior angles are congruent.})$$

$$60^\circ = \angle 2 + x^\circ \quad (\text{Triangle exterior angle theorem})$$

$$60^\circ = 45^\circ + x^\circ \quad (\text{Replace } \angle 2 \text{ with } 45^\circ)$$



We get

$$x^{\circ} = 15^{\circ}$$

$$x = 15$$

$$\text{So } x = 15, y = 60^{\circ}$$

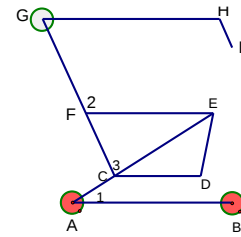
Q 10:

Given the figure of a baby stroller as shown in the figure.

$\overline{AB} \parallel \overline{CD} \parallel \overline{EF} \parallel \overline{GH}$, and $\overline{CG} \parallel \overline{IH}$.

If $\angle 1 = 25^{\circ}$, $\angle 2 = 110^{\circ}$.

Find the measure of $\angle 3$ and $\angle H$.



ANSWER:

Sol:

$$\angle ECD = \angle 1 = 25^{\circ} \quad (\overline{AB} \parallel \overline{CD}, \text{ congruent corresponding angles.})$$

$$\angle 2 = \angle ECD + \angle 3 \quad (\overline{CD} \parallel \overline{EF}, \text{ congruent corresponding angles.})$$

$$\text{Then } 110^{\circ} = 25^{\circ} + \angle 3$$

$$\angle 3 = 85^{\circ}$$

Furthermore,

$$\angle 2 + \angle G = 180^{\circ} \quad (\overline{EF} \parallel \overline{GH}, \text{ consecutive interior angles are supplementary.})$$

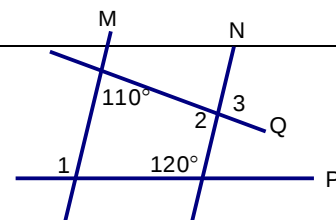
$$\angle H + \angle G = 180^{\circ} \quad (\overline{CG} \parallel \overline{IH}, \text{ consecutive interior angles are supplementary.})$$

$$\text{So } \angle 2 = 180^{\circ} - \angle G = \angle H$$

$$\text{Then } \angle H = \angle 2 = 110^{\circ}$$

Questions only:

Q 1:



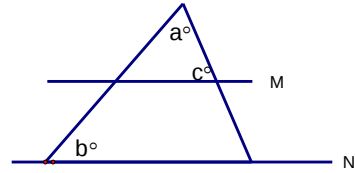
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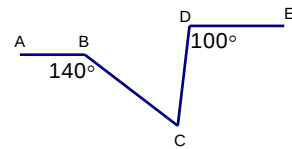
Find the value of c



Q 3:

$\overline{AB} \parallel \overline{DE}$, $\angle B = 140^\circ$, $\angle D = 100^\circ$.

Find the measure of $\angle C$.



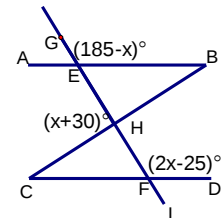
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Find

(3) The value of x

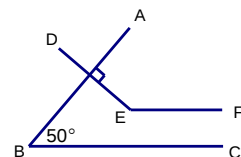
(4) The measure of $\angle C$.



Q 5:

As shown in the figure, $\overline{AB} \perp \overline{DE}$, $\overline{BC} \parallel \overline{EF}$, and $\angle B = 50^\circ$.

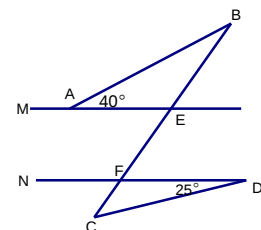
Find the measure of $\angle E$.



Q 6:

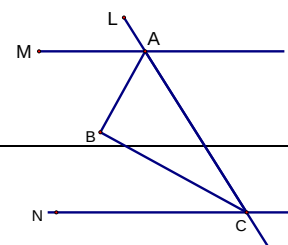
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Given lines M and N are parallel. Line L is a transversal and intersects lines M and N at points A and C respectively.



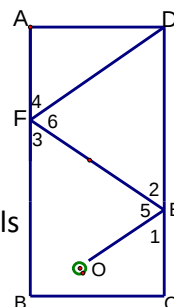
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Q 8:

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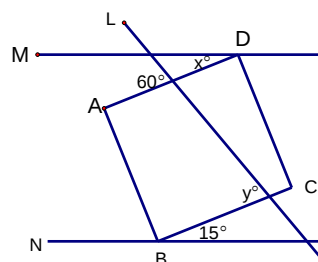


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Q 10:

Given the figure of a baby stroller as shown in the figure.

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If $\angle 1 = 25^\circ$, $\angle 2 = 110^\circ$.

Find the measure of $\angle 3$ and $\angle H$.

