Equations of lines

Different forms of equations of lines

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What are equations of lines(直線方程式）？

Review:

- What does the graph of the equation $2x + y - 5 = 0$ on the xy-coordinate look like?
- Could you provide other examples like this?
- What is the general form for an equation of a line?

The general form of the equation of the line is $ax + by + c = 0$ where a, b, c are constants.

使用建議	
教學活動安排	1.用提問複習國中學過的先備知識–二元一次方程式的圖形是 $ax+by+c=0$ 2.教師可依學生程度決定增加或刪減問題。
英文提問 / 開場	Hi everyone, today our topic is the equations of lines. We've covered this topic before, so let's take a moment to review and refresh our understanding. • What does the graph of the equation $2x+y-5=0$ on the xy–coordinate look like? • Could you provide other examples like this? • What is the general form for an equation of a line? So, the general form of equations of lines is $ax+by+c=0$ Now that we've learned how to find the slope of a line, let's take it a step further and learn how to write the equation of a line using its slope. Let’s begin the investigation. Give it a try and see what you can discover.

The point-slope form

- Investigation
- a. Sketch the line L passing through the point (2,1) with a slope of 2.
- b. List five points on the line.
- c. Please share with your partners some ways you find points.
- d. Given any arbitrary point (x, y), write an equation to check that the points pass through the line L. (Hint: Try to use the slope formula.)
- e. State the reason that the equation is the line L's equation.

How can we/you find the equations of the line?

- Given the slope and a point on a line, how can we/you find the equation of the line?
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Point-Slope Form of the Equation of a Line

The equation of the line with slope m passing through the point (x, y) is

$$y - y_1 = m(x - x_1)$$

More examples

- Find the following equations of lines in point-slope form and standard form.
- a. Passing through the point (1,4) with a slope of -3.
- b. Passing through the point (0,6) with a slope of $\frac{1}{2}$.
- c. Passing through the point (-2,1) with a slope of $-\frac{3}{2}$.

使用建議	
教學活動安排	1. 透過此探究希望學生能透過斜率公式去列出方程式 2. 問題a.教師可依學生程度要學生畫在有方格或沒方格的坐標平面上 3. 問題b.跟c. 教師看情況可以讓學生討論多種方式並討論哪種方式較快速或是能推到任何情況都適用（也許學生會說數格子最快，那教師可以反問如果斜率是根號2那種無理數呢？） 4. 完成探究後，帶學生回顧整個流程，給點跟斜率是如何導出直線方程式呢？ 5. 練習
英文提問 / 開場	Anyone wants to share the answers? Any other ways to list the points on the line? Which ways could apply to any given a point and a slope? Let’s wrap up what we’ve done to get the equation of the line. Excellent! Let’s pratice more examples. Have a go! We’ll check–in in five minutes.

Working with two points to get the equation of a line.

- Find the equation of the line that passes through points A$(1, -2)$and B$(4, -3)$.

Hint: First find the slope of the line.

More examples

- Find equations of the following lines.

a. Passing through points $(-1,4)$ and $(1,8)$

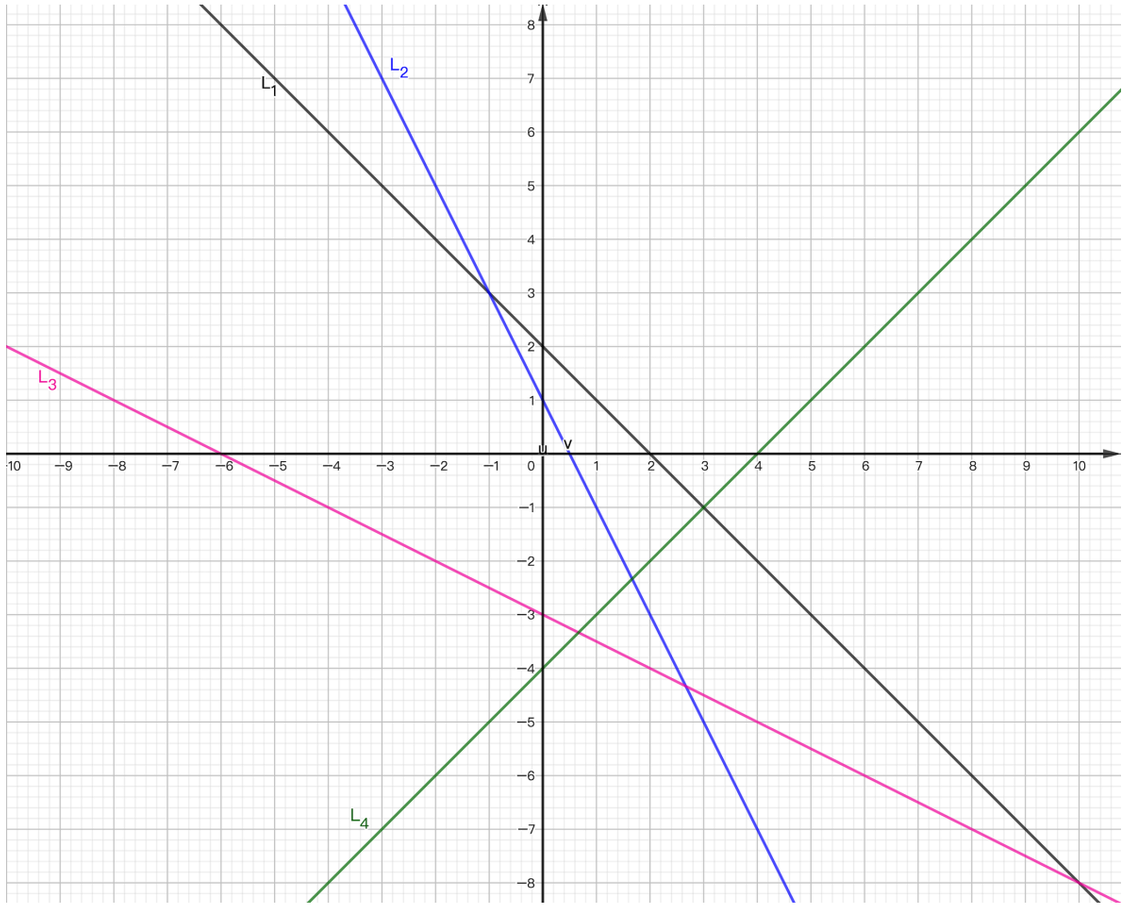
b. Passing through points $(-3,4)$ and $(1,4)$

c. Passing through points $(-3,-5)$ and $(-3,8)$

使用建議	
教學活動安排	1. 條件改成給定兩點如何求直線方程式？提示學生從如何求斜率去連結。 2. 練習c.為水平線，提醒學生水平線方程式國中學過。 3. 練習d提醒學生，沒有斜率就是鉛垂線，鉛垂線方程式國中學過。
英文提問 / 開場	We've learned that two points determine a line, and we also learned that we can get the slope from two points. So, if we are given two points, how do we find the equation of the line? Try starting with finding the slope, and then what comes next? The process is similar to how we derive the point–slope form of a line! Give it a try! Alright, let’s check the answer. Good job! Let’s get some more practice!

Other forms of an equation of a line: slope-intercept form

- Investigation



1. a. Identify the points at which line L_1

intersects the $x-$axis and$y-$axis .

b. Determine the slope of L_1.

c. Find the equation of line L_1.

Give the equation in point-slope form.

d. Rewrite the equation in the form $y = ax + b$

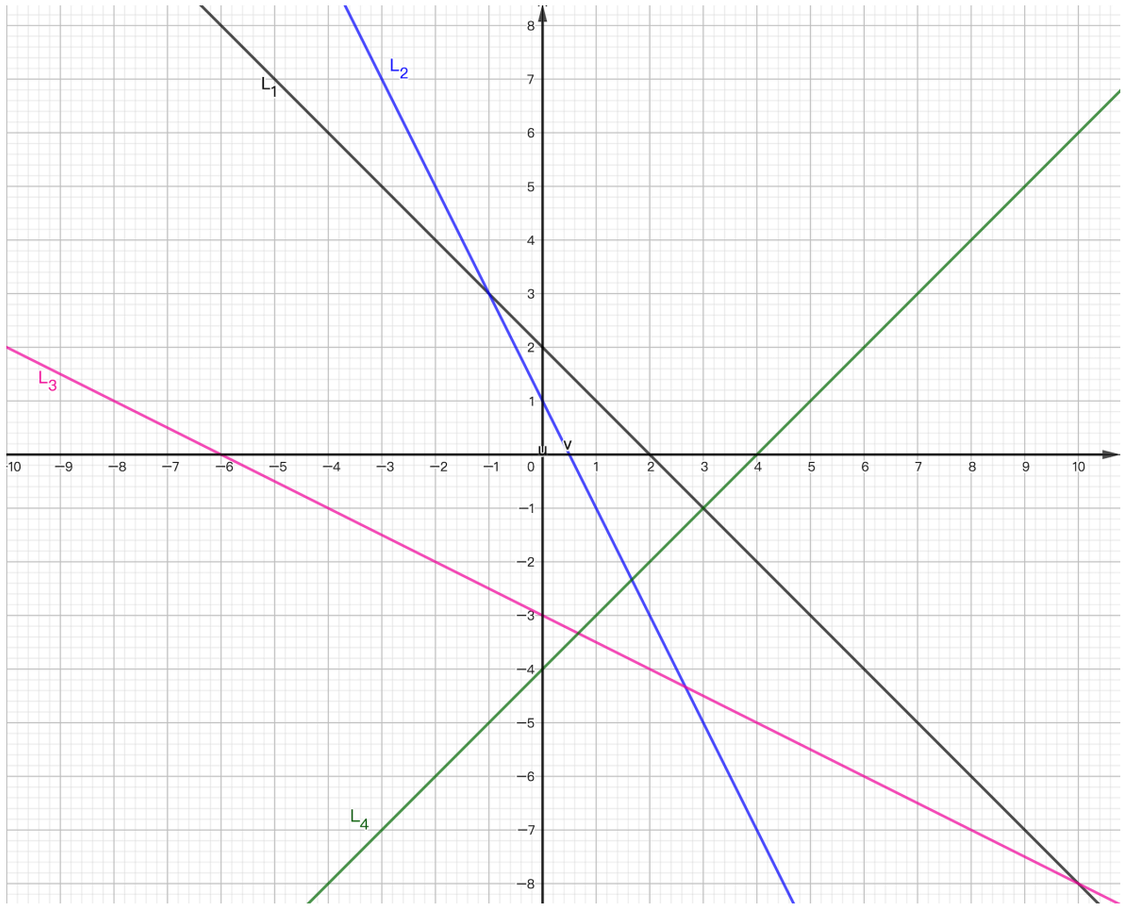
where $a \in \mathbb{Q}$ and $b \in \mathbb{Q}$

2.

a. Identify the points at which the line L_3 intersects the $x-$axis and $y-$axis.

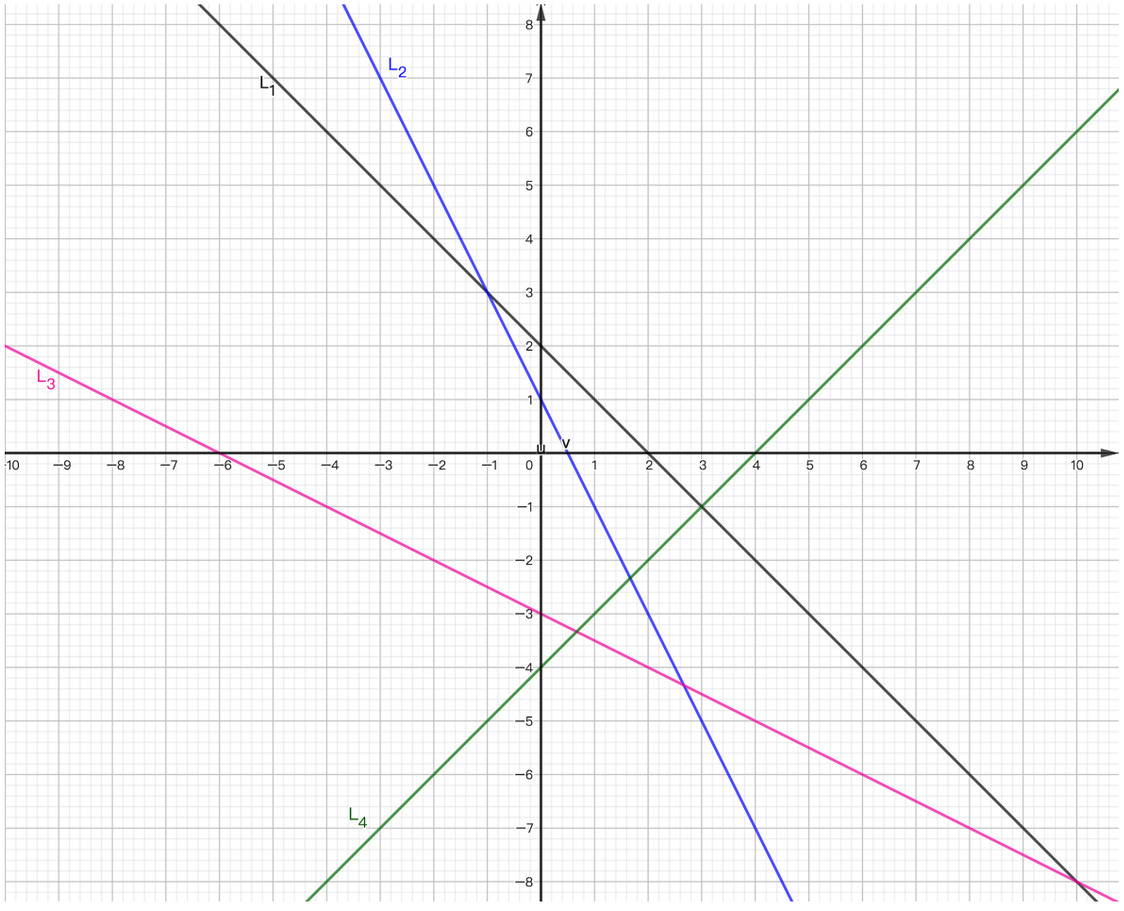
b. Determine the slope of L_3, and find the equation. Give the equation in point-slope form.

c. Rewrite the equation in the form $y = ax + b$ where $a \in \mathbb{Q}$ and $b \in \mathbb{Q}$



3. How do the values of a and b in your equations in the form $y = ax + b$ relate to the slope and intersection with axes? Write down your conjecture about a and b represent.

4. The equations of the lines L_2 and L_4 are $L_2 : 2x + y = 1$ and $L_4 : x - y - 4 = 0$. Rewrite the equations in the form $y = ax + b$, where where $a \in \mathbb{Q}$ and $b \in \mathbb{Q}$. Check whether you can confirm your conjecture from Q3 with the equations of lines L_2 and L_4. Write down your conclusions.



使用建議	
教學活動安排	透過探究讓學生發現直線方程式整理成 $y = ax + b$ 時，a 為斜率，b 為y截距。
英文提問 / 開場	Now let's investigate the general form $y = ax + b$ How does this relate to the slope? What do the coefficients a and b represent? Complete the following guiding questions to see what you can discover! Go and give it a try. Anyone want to share your thoughts? Alright, let's summarize what we've discovered.

Slope-intercept form

Equations of lines given in point-slope form or general form can be rearranged in the form below, which is called slope-intercept form:
$y = mx + k$, where m is the slope and k is the value of y for $x = 0$.

Intercepts: x-intercept and y-intercept

The point at which the graph intersects the x-axis is called the x-intercept, and the point at which the graph intersects the y-axis is called the y-intercept.
Note that the x-intercept has zero for its y-coordinate, and the y-intercept has zero for its x-coordinate.

Find the slope and y-intercept of the following equations.

a. $3x + 2y - 6 = 0$

b. $4x - y = 5$