Probability of Compound Events

✧ Words

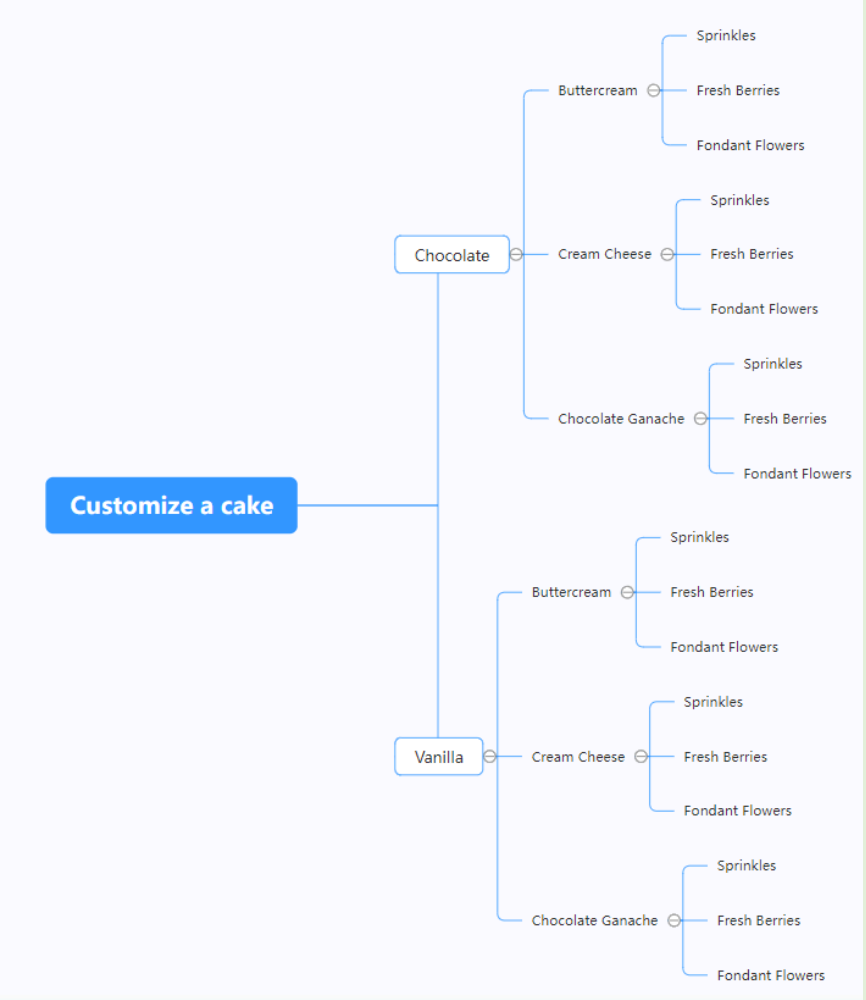
Some words have special meaning in Probability:

英文	中文	意義
Compound Events	複合事件	A compound event is an event that includes two or more simple events.
Tree diagram	樹狀圖	A tree diagram is a visual depiction of relationships that starts with a central node, or trunk.

✧ Example 1 Use a tree diagram.

A bakery offers customers different types of cake, frosting, and decorations for customizing a cake.
Types of cake: Chocolate or Vanilla
Frosting options: Buttercream, Cream Cheese, or Chocolate Ganache
Decorations: Sprinkles, Fresh Berries, or Fondant Flowers

A. Complete the tree diagram to represent the sample space.



B. The tree diagram shows that there are _____ equally likely outcomes in the sample space.

In this scenario, there are 2×3×3=18 equally likely outcomes in the sample space.

C. Find the probability P(Chocolate with Buttercream and Sprinkles) when selecting a cake at random.

To find this probability:

P(Chocolate with Buttercream and Sprinkles)

$$= \frac{\text{Number of outcomes where Chocolate, Buttercream, and Sprinkles are chosen}}{\text{Total number of equally likely outcome}}$$

$$= \frac{1}{18}$$

D. Find the probability of selecting a Chocolate cake with Buttercream frosting when selecting a cake at random.

To find this probability:

P(Chocolate and Buttercream)

$$= \frac{\text{Number of outcomes where Chocolate and Buttercream are chosen}}{\text{Total number of equally likely outcome}}$$

$$= \frac{3}{18} = \frac{1}{6}$$

麵包店為顧客提供不同類型的蛋糕、糖霜和裝飾品的選擇，以客製化蛋糕。選項有：

- 蛋糕類型：巧克力或香草
- 糖霜：奶油、奶油乳酪或巧克力甘納許
- 裝飾：糖粉、新鮮莓果或軟糖花

A. 完成樹狀圖來表示樣本空間。

B. 樹狀圖顯示樣本空間中有 _____ 種可能的結果。

C. 求隨機選擇蛋糕時選擇巧克力、奶油、糖粉的機率

D. 求隨機選擇蛋糕時選擇巧克力蛋糕和奶油糖霜的機率。

✧ Example 2 Use table.

Emma and Alex are playing a game with two spinners. Each spinner has numbers from 1 to 4. They want to find the probability that the sum of the two numbers they spin is 4.

A. Complete the table of sums:

2　　1	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

B. Observe from the table:

There are 3 possible outcomes with a sum of 4.
All outcomes in the sample space are equally likely.

C. The probability of spinning a sum of 4:

$$P(\text{sum of } 4) = \frac{\text{Number of equally likely outcomes that add to } 4}{\text{Total number of equally likely outcomes in the sample space}}$$

$$= \frac{3}{16}$$

D. Determine another possible outcome in the sample space that has the same probability as that of spinning a sum of 4.

$$P(\text{sum of } 4) = \frac{3}{16} = P(\text{sum of } 6)$$

✧ Example 3 Use organized lists.

Olivia is drawing cards from a hat that contains three cards labeled X, Y, and Z. Each card has an equal chance of being drawn. She draws one card at a time, records the result, and replaces it before the next draw. What is the probability that Olivia draws the X-card 3 times in a row?

A. Complete the organized lists to represent the sample space:

X on the first draw			Y on the first draw			Z on the first draw		
X	X	X	Y	X	X	Z	X	X
X	X	Y	Y	X	Y	Z	X	Y
X	X	Z	Y	X	Z	Z	X	Z
X	Y	X	Y	Y	X	Z	Y	X
X	Y	Y	Y	Y	Y	Z	Y	Y
X	Y	Z	Y	Y	Z	Z	Y	Z
X	Z	X	Y	Z	X	Z	Z	X
X	Z	Y	Y	Z	Y	Z	Z	Y
X	Z	Z	Y	Z	Z	Z	Z	Z

B. The organized lists show that there are ___ equally likely outcomes in the sample space.

$3 \times 3 \times 3 = 27$

C. What is the probability that Olivia draws the X-card 3 times in a row?

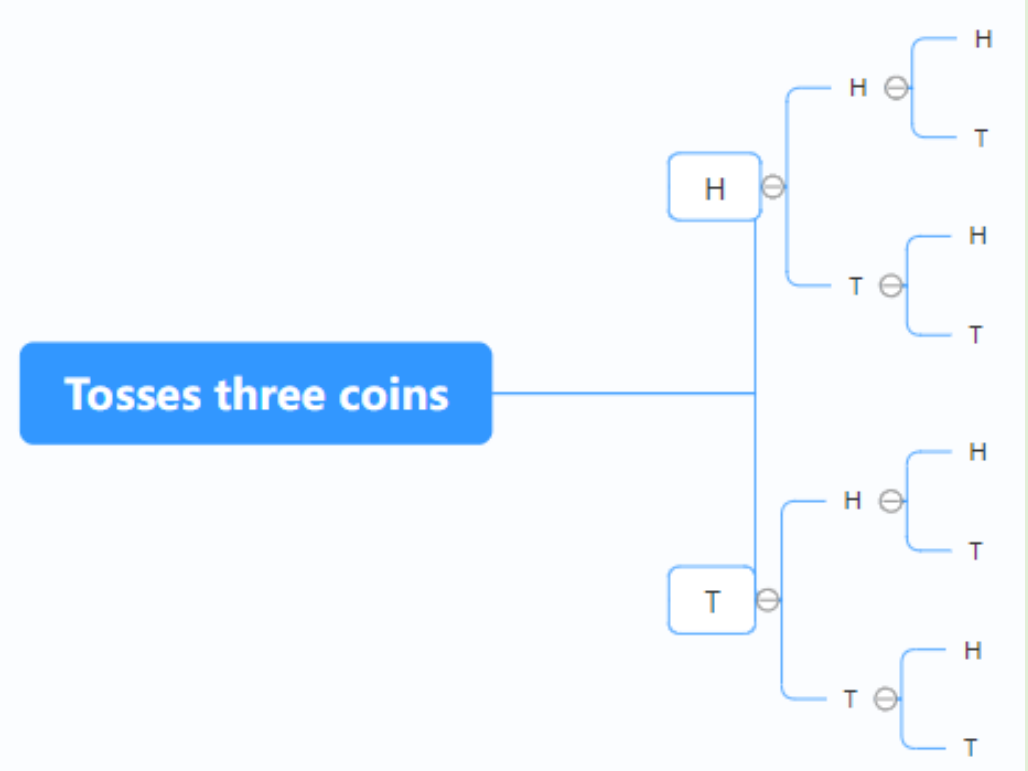
$$P(\text{X,X,X}) = \frac{\text{Number of equally likely outcomes in the event (X,X,X)}}{\text{Total number of equally likely outcomes in the sample space}}$$

$$= \frac{1}{27}$$

✧ Example 4 Toss three coins.

Sophia, Liam, and Emma each flip a coin. Each coin can land either heads (H) or tails (T) with an equal probability.

A. Complete the tree diagram to show the sample space for how the coins can land:



B. Probability that exactly two coins land heads up:

To find this probability, we count the number of outcomes where exactly two coins show heads.

Outcomes where exactly two coins are heads:

- HHT, HTH, THH

Total outcomes in the sample space = 8

$$P(\text{exactly two heads}) = \frac{3}{8} = 0.375 = 37.5\%$$

C. Probability that exactly two coins land tails up:

Similarly, we count the outcomes where exactly two coins show tails.

Outcomes where exactly two coins are tails:

- TTH, THT, HTT

$$P(\text{exactly two tails}) = \frac{3}{8} = 0.375 = 37.5\%$$

This probability is the same as the probability of exactly two heads.

D. Probability that exactly three coins land tails up:

There is only one outcome where all three coins show tails: TTT.

$$P(\text{exactly three tails}) = \frac{1}{8} = 0.125 = 12.5\%$$

E. Event that has the same probability as three tails:

An event that has the same probability as three tails (0.125 or 12.5%) could be the event where exactly three coins land heads up (HHH).

F. Compare those probabilities:

- Probability of getting at least one head:

- $P(\text{at least one head}) = 1 - P(\text{no heads}) = 1 - \frac{1}{8} = \frac{7}{8}$

- Probability of getting at least two heads:
  - $P(\text{at least two heads}) = \frac{4}{8} = \frac{1}{2}$

The probability of getting at least one head is higher than the probability of getting at least two tails.

*本教材的機率相關用詞有部分屬於臺灣高中才會介紹的內容，而國外有許多教材則會在國中階段即介紹這些用詞，老師們可以斟酌使用。