Basic trigonometry

I. Key mathematical terms

Terms	Symbol	Chinese translation
trigonometric ratio		
sine		
cosine		
tangent		
hypotenuse		
adjacent		
Opposite		

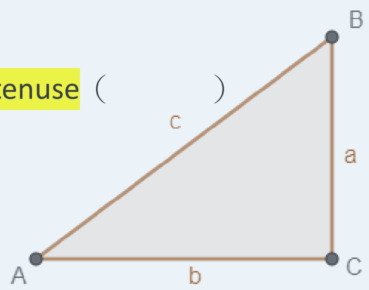
II. Acute-angled trigonometric ratios

Trigonometric ratios are vital in fields like architecture, engineering, and navigation. They help calculate angles, distances, and structural loads, ensuring precise designs and measurements. To better understand these, we'll start with the acute-angled trigonometric ratios. These ratios are specifically used for acute angles between 0° and 90°.

Let’s take a look at the following graph:

Trigonometric rations of acute angles

Hypotenuse (   )



Opposite (   )

Adjacent (   )

$$\sin\theta = \frac{a}{c} = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{b}{c} = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{a}{b} = \frac{\text{opposite}}{\text{adjacent}}$$

Now, let's apply the conclusions from the above figure and try the following example.

Example 1

In the triangle ABC, for $\angle C = 90°$, $\overline{AC} = 4$, $\overline{BC} = 5$.

(1) Find the values of $\sin A,\ \cos A,\ \tan A$

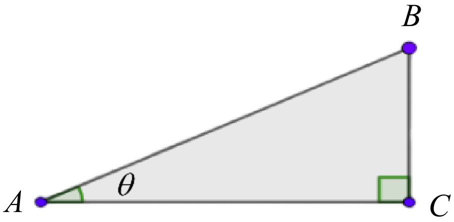
(2) Find the values of $\sin B,\ \cos B,\ \tan B$

(Compare your answer in (1) and (2). What do you notice? What is the relationship between them?)

Example 2

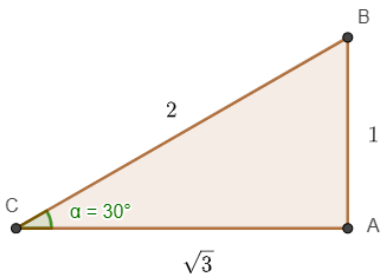
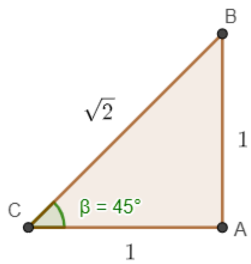
We know that $\sin\theta = \frac{3}{5},\ \cos\theta = \frac{4}{5}$.

Find the side length $\overline{AB},\ \overline{AC}$ of the triangle on the right.



Common values of $\sin x$, $\cos x$, $\tan x$

In junior high school, we learned about two special right-angled triangles: the $30° - 60° - 90°$ right triangle and the $45° - 45° - 90°$ isosceles ( ) right triangle. It is important to memorize the side rations of these two triangles. This way, when we need to calculate the trigonometric ratio for $30°,\ 45°,\ 60°$, we can use them directly.

	
$30° - 60° - 90°$ right triangle	$45° - 45° - 90°$ isosceles right triangle
side length ratio $1 : \sqrt{3} : 2$	side length ratio $1 : 1 : \sqrt{2}$

Example 3

Complete the following table by using the diagram on the previous page:

θ	$30°$	$45°$	$90°$
$\sin\theta$			
$\cos\theta$			
$\tan\theta$			

Example 4

θ is an acute angle and satisfy $\sin\theta = \frac{12}{13}$. Find the value of $\cos\theta$ and $\tan\theta$.

(Hint: You can draw a right-angled triangle on your own.)

Example 5

The length of the hypotenuse of a right-angled triangle is $10+3\sqrt{2}$ and one of the legs(邊) has length $3+5\sqrt{2}$. Determine the angles in the triangle.

III. Trigonometric identities

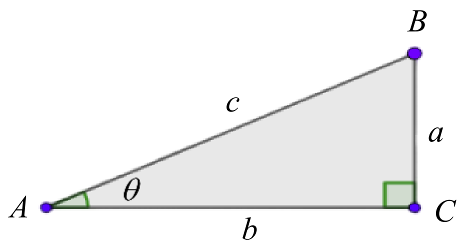
Through the previous examples, you may have noticed certain relationships between trigonometric ratios. Next, we will introduce the relationships between sine, cosine, and tangent. By understanding these relationships, we can solve problems more efficiently.

Acute-angled trigonometric identities

1. Quotient identity/relation: $\tan\theta = \frac{\sin\theta}{\cos\theta}$
2. Square identity/relation: $\sin^2\theta + \cos^2\theta = 1$
3. Complement identity/relation: (1) $\sin(90° - \theta) = \cos\theta$ (2) $\cos(90° - \theta) = \sin\theta$

The relationships mentioned above are correct for both acute angles and general angles which will be discussed later. However, in this unit, we will focus on the explanation of acute angles.

You can prove the identities quite easily using a right-angled triangle.



1. Quotient identity: $\dfrac{\sin\theta}{\cos\theta}=\dfrac{a}{c}\div\dfrac{b}{c}=\dfrac{a}{c}\times\dfrac{c}{b}=\dfrac{a}{b}=\tan\theta$
2. Square identity: $\sin^2\theta+\cos^2\theta=\dfrac{a^2}{c^2}+\dfrac{b^2}{c^2}=\dfrac{c^2}{c^2}=1$ (Using Pythagoras' Theorem)
3. Complement identity: Use the same triangle with different orientations. You can try to prove it on your own.

Example 6

Show that $(\sin\theta+\cos\theta)(\sin\theta-\cos\theta)=2\sin^2\theta-1$

(Hint: Use the square identity)

Example 7

Use the trigonometric identities to fill in the blanks:

1. $\dfrac{\sin 31°}{\cos 31°}=\tan(\quad\quad)$
2. $\sin 18°=\sin(90°-\quad\quad)=\cos(\quad\quad)$
3. $\cos 80°=\cos(90°-\quad\quad)=\sin(\quad\quad)$
4. $\sin^2 23°+\cos^2 23°=(\quad\quad)$
5. $\dfrac{\sin 53°}{\sin 37°}=\tan(\quad\quad)$

Through the basic examples above, you should have a general understanding of the properties of trigonometric ratios. Next, let's look at a few applied problems with variations.

Example 8

In a right-angled triangle ABC, $\angle C = 90°$ and $\tan A = a$ is given. Express the values of $\sin A$, $\cos A$ in terms of a.

Example 9

θ is an acute angle and satisfy $\sin\theta - \cos\theta = \frac{\sqrt{5}}{2}$. Find the value of

(1) $\sin\theta\cos\theta$ (2) $\sin\theta + \cos\theta$

<資料來源>

1. Acute-angled trigonometric ratio

https://byjus.com/maths/trigonometric-ratios/

https://www.mathsisfun.com/algebra/trigonometry.html

https://www.mathsisfun.com/sine-cosine-tangent.html

2. Trigonometric identities

https://byjus.com/maths/trigonometric-identities/

https://www.cuemath.com/trigonometry/trigonometric-identities/

3. 南一書局數學（二）

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