Topic：Sets

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Warm-up

A survey was conducted on the drinking habits of all 40 students in the class. It was found that 30 students prefer 'half sugar' (half the normal amount of sugar), and 20 students prefer 'no ice' (no ice added).

1. **How can we organize this data so it's easier to understand? What kind of chart or graph could we use?**
2. **What is the maximum and minimum number of students in the class who prefer both 'half sugar' and 'no ice'?**

使用建議	
教學活動安排	透過暖身問題，引起學習動機。
英文提問 / 開場	Today, we're going to learn about a new math tool called sets. Sets can help us organize and analyze information more easily. But before we dive into the details, let’s start with a question. What does this question ask about? Does anyone explain what the situation is about this question? Imagine our class of 40 students. Some of you prefer drinks with 'half sugar,' while others prefer 'no ice.' We can think of the students who like 'half sugar' as one group, and those who like 'no ice' as another group. Now, do you think there could be some students who belong to both groups? In other words, could some of you like both 'half sugar' and 'no ice' in your drinks?" What kind of graph or diagram could help us visualize these two groups and any overlap between them? To answer these questions, we'll first need to understand today’s topic, sets. Once we’ve learned about sets, we’ll come back to solve this question. So, let’s explore how sets can help us!

Sets (集合）

-Definition

A **set** is a collection of numbers or objects.
Each object is called an **element(元素）** or **member** of the set.

-Notations and the ways to express the sets and the elements of the sets.

For example, Let B represents the set of the K-pop group BLACK PINK then

B={Jisoo,Jennie,Rosé,Lisa} (List all the members of BLACKPINK)(roster notation)

or B={$x|x \in \mathrm{BLACKPINK}$} (set builder notation)

-Two sets are equal if they contain exactly the same elements.

Set notations

- $\in$ means “is an element of” or “is in”
- $\notin$ means “is not an element of” or “is not in”
- $n(A)$ means “the number of elements in the set A”

A **set** is a collection of numbers or objects.
Each object is called an **element(元素）** or **member** of the set.

Two sets are **equal** if they contain exactly the same elements.

Example 1.

Let P be the set of letters in the word AMATEUR
and Q be the set of letters in the word TEAM.

a. List the elements of P and Q
b. Find n(P) and n(Q)

Example 2.

The definition of rational numbers is any number that can be expressed as the ratio of two integers, where the denominator is not zero.

Let Q be the set of all rational numbers, use the set builder notation to write the set Q and find n(Q).

使用建議	
教學活動安排	介紹集合的定義及符號及表示法，並用兩個例子讓學生練習。其中例子一可銜接後續介紹子集合的概念
英文提問 / 開場	Let's start with the definition of the sets. Does anyone want to read it for us? Good! There are two ways to express the sets, let's take the BLACK PINK for example. Let B represent the set of the K-pop group BLACK PINK then We list all the members of BLACKPINK in a curly bracket like this B={Jisoo, Jennie, Rosé, Lisa}. This is called the roster notation. Or we can write like this B=$\{x \mid x \in \mathrm{BLACKPINK}\}$(B is the set x such that x belongs BLACKPINK). This is called the set builder notation. Let's look into the notations we used to write a set. Now it's your turn to practice, do the example 1 and 2. We'll check-in in five minutes.

Different types of sets.

- Finite set(有限集合)、Infinite set(無限集合)

Set A is a finite set if $n(A)$ has a particular defined value.
If A has an endless number of elements, we say it is an infinite set.

- Subset (子集合)

A is a **proper subset** of B if every element of A is also an element of B, but $A \neq B$.
We write $A \subset B$.
Set A is a **subset of** set B if every element of A is also an element of B. We write $A \subseteq B$.

Different types of sets.

- Empty set (空集合)

The empty set ∅ or { } is a set which contains no elements.

-The empty set is a subset of all other sets

- Universal set (宇集)

The universal set U is the set of all elements we are considering.

Example 3

If U is the set of the K-pop group BLACK PINK then

U={Jisoo,Jennie,Rosé,Lisa}

a. List all the subsets of set U.
b. Given that A={Jisoo,Lisa},B={Jisoo,Lisa,Rosé}. Decide whether the statement is true or false

i. Jennie $\in A$ ii. Lisa $\in B$ iii. $A \subseteq B$ iv. $A \subset B$

使用建議	
教學活動安排	介紹子集合等不同集合的定義
英文提問 / 開場	For example 1, you can see all elements in set Q are also in set P. We can say the set Q is included in the set P. It means the set Q is a subset of the set P and is denoted as $Q \subseteq P$. So let's look at the definition of subsets and different types of sets. Does anyone want to read it for us? Any questions? It's your turn to do example 3. We'll check-in in five minutes. Good. Let's go back to the warm-up question. In that question, the class is the universal set, and there are two subsets, one is the students who like 'half sugar' and one who like 'no ice'. There could be some students who belong to both sets. This brings the concept of intersections of sets.

使用建議	
教學活動安排	介紹交集、聯集、補集
英文提問 / 開場	Let’s go back to the warm-up question. In that question, the entire class of 40 students represents the universal set, which includes everyone we're considering. Within this universal set, we have two smaller groups, or subsets. One subset is made up of students who prefer 'half sugar,' and the other subset is students who prefer 'no ice.' These are two separate groups within the larger class. Now, some students might belong to both subsets—they prefer both 'half sugar' and 'no ice.' This overlap is called the intersection of the two sets. The intersection represents the students who are in both groups. On the other hand, if we combine all the students who prefer 'half sugar' with all the students who prefer 'no ice'—including those who might like both—we call this the union of the two sets. The union includes all students who prefer either 'half sugar,' 'no ice,' or both. By understanding the concepts of intersection and union, we can better organize and analyze information like this. Let’s check the definition of the union and intersection of sets and see more examples.

Intersection（交集）

U={x|x∈BLACK PINK}

A is a subset of U , A={Jisoo,Lisa},

B is a subset of U, B={Jisoo,Lisa,Rosé} ,we can find Lisa is in both set A and set B.

The intersection of two sets A and B is the set of elements that are in both set A and set B.
The intersection of sets A and B is written $A \cap B$.(We say A intersection B.)

Two sets A and B are disjoint or mutually exclusive if they have no elements in common.
In this case $A \cap B = \emptyset$.

Union（聯集）

The union of two sets *A* and *B* is the set of elements that are in either sets *A* and *B* is written A∪B. (A union B)

For example,

A={Jisoo,Lisa,Rosé},

B={Jin,RM} ,then A∪B={Jisoo,Lisa,Rosé,Jin,RM}

Complement of a set (集合的補集)

The complement of a set A is the set of all elements of U that are **not** elements of A.
The complement of A is written A'.

For example, $U=\{x|x\in$our classmates$\}$.

A is the set of k-pop fans , $A'=?$

How can we show the idea(concept) of intersection in a diagram?

使用建議	
教學活動安排	介紹文氏圖
英文提問 / 開場	A Venn diagram is a helpful tool that allows us to visualize the relationships between different sets and understand their connections more clearly. Did you know that Vice President Kamala Harris often uses Venn diagrams to explain her ideas? She finds them effective for helping people see how different concepts or groups relate to each other, making complex topics easier to grasp. In math, we can use Venn diagrams to show how sets overlap, combine, or differ from one another. Let's watch a short clip that explains the details of Venn diagrams and how they work. (After watching the clip.)To make sure you fully understand how Venn diagrams represent different sets and their relationships, complete the following exercise. We'll check-in in five minutes.

Venn diagrams

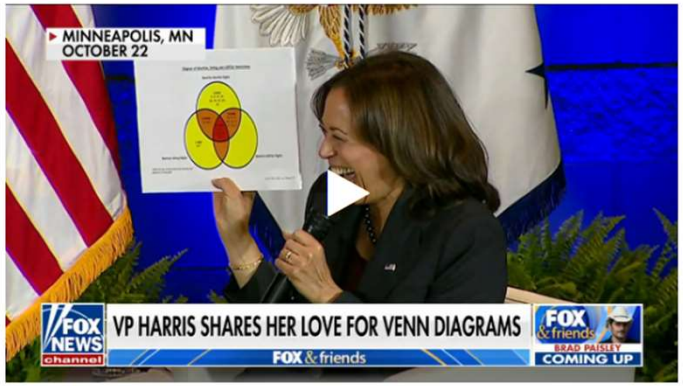
KAMALA HARRIS

Kamala Harris mocked for repeatedly sharing love of Venn diagrams: 'This is just bizarre'

'When you peak in third and fourth grade and become vice president, we're doomed,' Brian Kilmeade said

By Elizabeth Heckman · Fox News

Published February 24, 2023 2:00pm EST

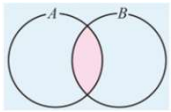
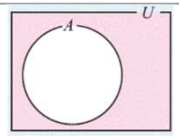


Kamala Harris again professes her love for Venn diagrams

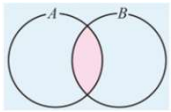
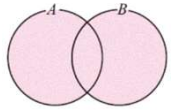
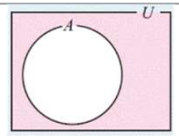
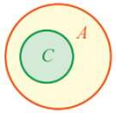
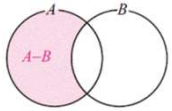
Vice President Kamala Harris explained again at the White House Thursday why she loves Venn diagrams.

https://youtu.be/KoS1y8xridY?si=gsNQzgrO72uF6wln

Exercise. Fill the following blanks.

Notation	Definition	Diagram(the shaded region)
$A \cap B$	$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$	
$A \cup B$	$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$	
A'		
$C \subseteq A$	C is a subset of A	
$A - B$	$A - B = \{x \mid x \in A \text{ and } x \notin B\}$	

Exercise.

Notation	Definition	Diagram(the shaded region)
$A \cap B$	$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$	
$A \cup B$	$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$	
A'	$A' = \{x \mid x \notin A\}$	
$C \subseteq A$	C is a subset of A	
$A - B$	$A - B = \{x \mid x \in A \text{ and } x \notin B\}$	

使用建議	
教學活動安排	完成開場暖身問題，並帶出取捨原理。
英文提問 / 開場	Now, let’s return to our warm-up question and solve it using what we've learned. Let's place the information in a Venn diagram, showing the students who prefer 'half sugar' and those who prefer 'no ice. What do you think the maximum and minimum number of students who prefer both could be? How might this be related to the total class size of 40 students?Great! This leads us to an important concept called the principle of inclusion and exclusion. It’s a property that connects the union and intersection of sets, helping us find the total number of elements in combined sets. Let's explore this property together and see how it can help us solve problems like this."

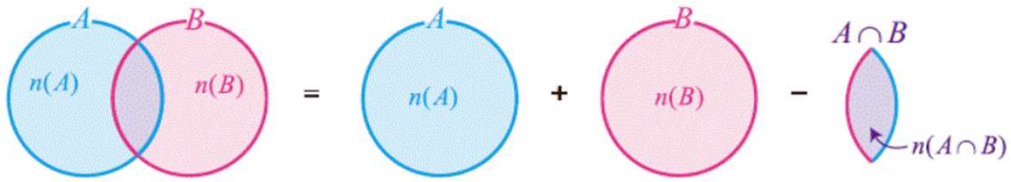
Exercise. Place this information on a Venn diagram

A survey was conducted on the drinking habits of all 40 students in the class. It was found that 30 students prefer 'half sugar' (half the normal amount of sugar), and 20 students prefer 'no ice' (no ice added).

The principle of inclusion-exclusion(取捨原理)

The union and intersection of two sets have the following relationship, which we call the principle of inclusion and exclusion.

$$n(A \cup B)=n(A)+n(B)-n(A \cap B)$$



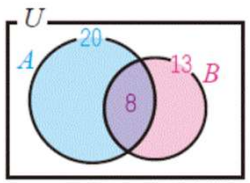
In particular,when$A \cap B=\varnothing$,that is $n(A \cap B)=0$, we have $n(A \cup B)=n(A)+n(B)$.

Example 4

In a class of 35 students, 20 students like playing basketball, 13 students like playing baseball, and 8 students like both sports. How many students in the class like either basketball or baseball?

Solution:

Let A and B represent the sets of students who like basketball and baseball, respectively.
From the information given, we know that $n(A) = 20$, $n(B) = 13$, and $n(A \cap B) = 8$.
Using the inclusion-exclusion principle, we get:



$n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$= 20 + 13 - 8$
$= 25.$

Thus, a total of 25 students like either basketball or baseball.

Exercise.

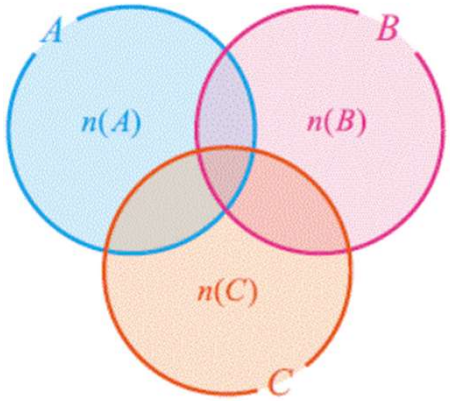
A survey was conducted on the drinking habits of all 40 students in the class. It was found that 30 students prefer 'half sugar' (half the normal amount of sugar), and 20 students prefer 'no ice' (no ice added).

What is the maximum and minimum number of students in the class who prefer both 'half sugar' and 'no ice'?

使用建議	
教學活動安排	給學生圖形提示去嘗試推出3個集合的取捨原理。
英文提問 / 開場	The principle of inclusion and exclusion can also be extended to three sets. Please refer to the diagram to complete the inclusion-exclusion principle for the three events A, B, and C below. Have a go and give it a try. We'll check-in in five minutes. Anyone wants to share your work?"

Exercise.

The following diagram uses a Venn diagram to represent the possible relationships between events A, B, and C. Please refer to the diagram to complete the inclusion-exclusion principle for the three events A, B, and C below.



參考資料

1. https://www.youtube.com/watch?v=LumU80IN748
2. Mathematics: Core Topics SL