PH Wu

Permutations I – Test

Class: ______ Number: _____ Name: _____

Part A: True/False (10 pts each)

Instruction: <u>Mark "T" for true or "F" for false for the following statements.</u>

()1. 3! represents "3 factorial" and is equal to $3 \times 2 \times 1$.

()2. There are $\frac{6!}{2!2!2!}$ ways to arrange PAPAPA in a single line.

Part B: Multiple Choice (10 pts each)

Instruction: For each question below, select the best answer from the provided options. There is at least one correct answer for each question.

() 1. Which of the following statements are correct?

- A) 0! = 0
- B) $P_0^5 = 1$
- C) $P_2^6 = P_4^6$

D)
$$P_3^4 = \frac{4!}{3!}$$

E) $\frac{P_4^6}{6} = P_3^5$

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) 2. In the grid street diagram on the right, how many different shortcuts are there from point A to

point B, if you can only move right or up?

A)
$$\frac{5!}{3!2!} \times \frac{4!}{2!2!} + 2$$

B) $\frac{6!}{3!3!} \times \frac{5!}{2!3!} + 2$
C) $P_3^5 \cdot P_2^4$
D) 62





Part C: Short Answer (10 pts each)

Instruction: Provide clear and concise answers to each of the questions.

 If you randomly choose one or more flags, which have 4 different colors, to hang on a flagpole(旗桿) for signaling, and the order of the flags matters, how many different signals can be created?

 Suppose you roll 5 fair, 4-sided dice one after the other. How many different possible outcomes are there?

- 3. The *pentatonic musical scale*(五聲音階) contains 5 notes in an octave(八度): C, D, E, G, and A. Lulu decides to look for new musical themes by playing random sequences of 3 notes from the pentatonic scale.
 - i) How many possible three-note sequences can be played?
 - ii) How many possible three-note sequences contain no Gs?

4. After graduation, Lulu invited her students to dinner, three girls and three boys, where they posed for a photo. One girl and one boy had an unpleasant experience and do not want to stand next to each other. Lulu must stand in the middle, and the three boys cannot all be on the same side. How many possible arrangements are there?

Part D: Problem Solving (10 pts each)

Instruction: Show all the necessary steps. Neatness counts. Be sure to include units of measurement where relevant.

As shown in the figure below, there is an equilateral triangular area on the wall with a side length of 3. There are several blue tiles with a side length of 2, red tiles with a side length of 1, and green tiles with a side length of 1. Completely fill the triangular area on the wall using these tiles, ensuring that at least one blue tile is included. How many different patterns can be created?

Ch4. Permutations I – Answer Key

Part A

- 1. T
- 2. F

Part B

- 1. BE
- 2. AD

Part C

1. One flag + two flags + three flags + four flags =

 $4 + 4 \times 3 + 4 \times 3 \times 2 + 4 \times 3 \times 2 \times 1 = 4 + 12 + 24 + 24 = 64$

- $2. \quad 4 \times 4 \times 4 \times 4 \times 4 = 1024$
- 3. i) $5 \times 4 \times 3 = 60$; ii) $4 \times 3 \times 2 = 24$
- 6!-2·3!·3!-4·2!·4!=456 (Total arrangements a boy and a girl stand together three boys on the same side.)

Part D



For each of the 5 small triangles, we have two color options: green and red.

 $2^5 + 2^5 + 2^5 = 96$

Ch4. Permutations I – Vocabulary

- 1. factorial 階乘
- 2. arrange 安排
- 3. shortcut 捷徑
- 4. outcome 結果