雙語教學主題(國中八年級下學期教材)三角形邊角關係及樞紐及其逆定理

Topic: TEST QUESTIONS FOR THE RELATIONSHIP BETWEEN SIDES AND ANGLES IN TRIANGLES PLUS HINGE THEOREM AND ITS CONVERSE

Vocabulary

CPCTC stands for *corresponding parts of the congruent triangles are congruent*		兩全等三角形的對應邊和對應角都對應相等	
trichotomy law	三一律	converse theorem	逆定理
measure	測量	segment	線段
triangle inequality theorem	三角形邊角關係定理	exterior angle theorem	外角定理
fundamental	基本的	opposite	相對的
true	正確的	false	錯誤的
exterior angle inequalitytheorem	外角大於任一內對角	converse of the hinge theorem	樞紐逆定理
absolute value	絕對值	the hinge theorem	樞紐定理
subtraction	減法	substitution	代替

(我個人很喜歡這個 CPCTC 的表示法，否則要寫很多中文字…)

老師們好，這是三角形邊角關係及樞紐及其逆定理的應用練習。這份教材的前半是題目加解答，後半是題目，方便老師們參考使用。老師們可以依據學生的需求選取適當的題目供學生做小測或是練習。

Q 1:

Which of the following could be the sides of a triangle?

A. 11,12,13 B. 3,4,5 C. 3,6,9

ANSWER:

Sol:

A. 11+12>13 23>13 true

11+13>12 24>12 true

12+13>11 25>11 true so 11,12,13 can be the sides of a triangle.

B. 3+4>5 7> 5 true

3+5>4 8>4 true

4+5>3 9>3 true so 3,4,5 can be the sides of a triangle.

C. 3+6>9 9> 9 false

3+9>6 12>6 true

6+9>3 15>3 true so 3,6,9 **can not** be the sides of a triangle.

YES: A, B

NO: C

Q 2:

The measures of the three segments from the least to the largest are x-1, x+2, and 3x-1 respectively. If they can form a triangle, please find the range of the values of x.

ANSWER:

Sol:

We know the order of these three side lengths, we can just simplify the process of applying the triangle inequality theorem by only doing

" the sum of the two shorter side lengths is greater than the longest" instead.

Therefore, we have

$$x-1+x+2>3x-1>|x-1-(x+2)|$$

$$\Rightarrow \quad 2x+1>3x-1>3$$

$$2x+1>3x-1 \quad \text{and} \quad 3x-1>3$$

$$x<2 \quad \text{and} \quad x>\frac{4}{3}$$

i.e. $\frac{4}{3}<x<2$#

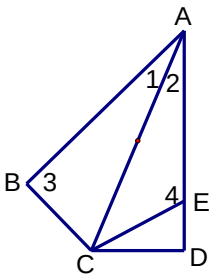
Reminder:

We need to check if the three side lengths are positive within the range of x.

Side lengths are always positive.

Q 3:

Given $\overline{AB} = \overline{AD}$, $\angle 1 = \angle 2$. Compare $\angle 3$ and $\angle 4$.



ANSWER:

Sol"

In ΔABC and ΔADC,

$\overline{AB} = \overline{AD}$ (Given)

$\angle 1 = \angle 2$. (Given)

$\overline{AC} = \overline{AC}$ (Reflexive property)

$\Delta ABC \cong \Delta ADC$, (SAS)

$\angle 3 = \angle D$ (CPCTC)

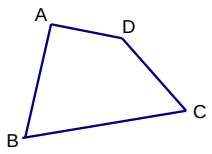
And in ΔCDE,

$\angle 4 > \angle D$ (Exterior angle inequality theorem)

$\angle 4 > \angle D = \angle 3$

$\Rightarrow$ $\angle 4 > \angle 3$#

Q 4:



Given quadrilateral ABCD, $\overline{AD}$ is the shortest side, and $\overline{BC}$ is the longest side. Compare the measure of $\angle A$ and $\angle C$ and please write down your reasons.

ANSWER:

Sol:

When we want to compare sides or angles, we need triangles for us to work on. So we create triangles in the first place.

Connect $\overline{AC}$ as shown in Figure 1.

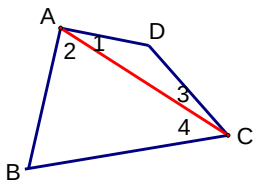


Figure 1

We have two triangles here, ΔABC and ΔACD.

We work on one triangle at a time because the triangle inequality theorem needs only to be discussed in one triangle.

We first look into ΔABC.

In ΔABC,

$\overline{BC} > \overline{AB}$ (Given $\overline{BC}$ is the longest side)

$\Rightarrow$ $\angle 2 > \angle 4$ (side-angle inequality theorem)

In ΔACD,

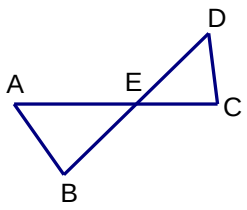
$\overline{CD} > \overline{AD}$ (Given $\overline{AD}$ is the shortest side)

$\Rightarrow$ $\angle 1 > \angle 3$ (side-angle inequality theorem)

Then $\angle A = \angle 1 + \angle 2 >> \angle 3 + \angle 4 = \angle C_{\#}$

Q 5:

As shown in the figure, $\overline{AC}=\overline{BD}$, $\overline{CE}=\frac{2}{5}\overline{AC}$



Point E is the midpoint of $\overline{BD}$.

Compare the measure of $\angle A$ and $\angle B$.

ANSWER:

Sol:

Since $\angle A$ and $\angle B$ are in the same triangle ABE, we only need to look at ΔABE.

In ΔABE,

$$\overline{AE}=\overline{AC}-\overline{CE}=\overline{AC}-\frac{2}{5}\overline{AC} \qquad \text{(Given)}$$

$$=\frac{3}{5}\overline{AC}$$

And $\overline{BE}=\frac{1}{2}\overline{BD}$ (Point E is the midpoint of $\overline{BD}$)

$$=\frac{1}{2}\overline{AC}$$

Therefore,

$$\overline{AE}=\frac{3}{5}\overline{AC}>\frac{1}{2}\overline{AC}=\overline{BE}$$

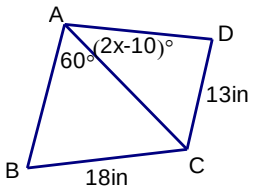
$\Rightarrow \overline{AE}>\overline{BE}$

$\Rightarrow \angle B>\angle A_{\#}$

Q 6:

In the figure on the right, $\overline{AB}=\overline{AD}$, $\overline{BD}=18in$, $\overline{CD}=13in$. $\angle BAC=60^\circ$, and $\angle CAD=(2X-10)^\circ$.

Find the range of the values of x.



ANSWER:

In ΔABC and ΔADC,

$\overline{AB}=\overline{AD}$, (Given)

$\overline{AC}=\overline{AC}$ (Reflexive property)

$\overline{BD}=18in>\overline{CD}=13in$

$\angle CAB>\angle CAD$ (The converse of the hinge theorem)

$\Rightarrow$ $2x-10<60$

$2x<70$

$X<35$ …① (Simplify)

However,

Any angle of a triangle must be greater than 0,

So $2x-10.>0$

$2x>20$

$x>5$ …② (Simplify)

From① and ②

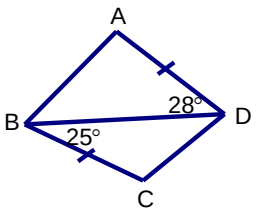
$5<x<35$#

Q 7:

Compare the sides and angles and fill in the blanks with a '>' or '<' symbol.

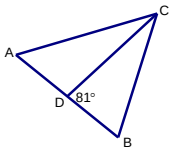
(1) As shown in the Figure on the right, $\overline{BC}=\overline{AD}$, $\angle ADB=28°$, and $\angle CBD=25°$.

Then $\overline{CD}$ _____ $\overline{AB}$



(2) In ΔABC, $\overline{AD}=\overline{BD}$, and $\angle BDC=81°$.

Then $\overline{AC}$ _____ $\overline{BC}$.



ANSWER:

Sol:

(1) In ΔADB and ΔCBD

$\overline{BC}=\overline{AD}$ (Given)

$\overline{BD}=\overline{BD}$ (Reflexive property)

$25°<28°$ (Given)

$\Rightarrow$ $\angle CBD<\angle ADB$

Then $\overline{CD}$ ___<___ $\overline{AB}$ (The hinge theorem)

(2) In ΔADC and ΔBDC,

$\overline{AD}=\overline{BD}$ (Given)

$\overline{CD}=\overline{CD}$ (Reflexive property)

$\angle BDC=81°$ (Given)

$\angle ADC=180°-81°$

$=99°$ ($\overline{AB}$ is a segment, a straight angle equals $180°$)

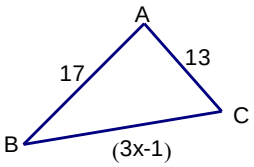
$\angle ADC>\angle BDC$

Then $\overline{AC}$ ___>___ $\overline{BC}$ (The hinge theorem)

Q 8:

In ΔABC, $\overline{AB}=17$, $\overline{AC}=13$, $\overline{BC}=3x-1$, and $\angle A>\angle C>\angle B$.

Find the range of the values of x.



ANSWER:

Sol:

By the triangle inequality theorem,

$17-13< 3x-1<17+13$

$4<3x-1<30$

$5<3x<31$

$\frac{5}{3}<x<\frac{31}{3}$ …① (Simplify the inequality)

Furthermore, $\angle A>\angle C>\angle B$

$\Rightarrow$ $\overline{BC}>\overline{AB}$ (Triangle inequality theorem)

i.e. $3x-1>17$ (Substitution)

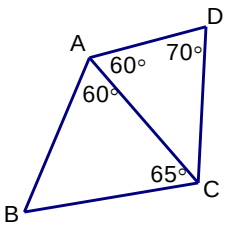
$x>6$ …②

From ① and ②,

$6<x<\frac{31}{3}$ #

Q 9:

In quadrilateral ABCD, $\angle BAC=\angle DAC=60^\circ$, $\angle BCA=65^\circ$, and $\angle ADC=70^\circ$. List the lengths of the sides in quadrilateral ABCD In order.



ANSWER:

Sol:

Because In ΔCAB and ΔCAD, $\overline{AC}$ is the common side. We need to build up the relationships of sides related to $\overline{AC}$.

In ΔCAB,

$\angle B=180^\circ-\angle BAC-\angle BCA$ (The interior angle sum theorem)

$=180^\circ-60^\circ-65^\circ$,

$=55^\circ$

$55^\circ<60^\circ<65^\circ$

$\Rightarrow \overline{AC}<\overline{BC}<\overline{AB}$ …①

Similarly,

In ΔCAD,

$\angle ACD=180^\circ-\angle CAD-\angle CDA$ (The interior angle sum theorem)

$=180^\circ-60^\circ-70^\circ$

$=50^\circ$

$50^\circ<60^\circ<70^\circ$

$\Rightarrow \overline{AD}<\overline{CD}<\overline{AC}$ …②

From ① and ②, we get

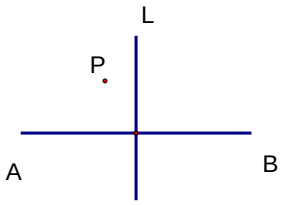
$\overline{AD}<<\overline{AC}<\overline{BC}<\overline{AB}$

So these four sides from the shortest to the longest are:

$\overline{AD}$, $\overline{CD}$, $\overline{BC}$, $\overline{AB}$ #

Q 10:

Line L is the perpendicular bisector of $\overline{AB}$. Point P is on the left side of L. Prove $\overline{PA} < \overline{PB}$.

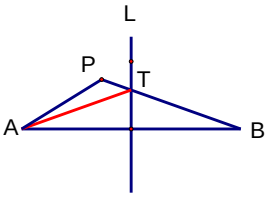


ANSWER:

Pf:

Connect $\overline{PA}$ and $\overline{PB}$. $\overline{PB}$ intersects line L at point T, as shown in Figure 1.

Connect $\overline{AT}$.



Line L is the perpendicular bisector of $\overline{AB}$ (Given)

$\overline{TA} = \overline{TB}$ (Property of perpendicular bisector.)

In ΔAPT,

$\overline{AP} < \overline{TA} + \overline{TP}$

$\Rightarrow$ $\overline{AP} < \overline{TB} + \overline{TP}$ (Substitution)

$\overline{PA} < \overline{PB}$. (Property of addition)

QUESTIONS ONLY:

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Which of the following could be the sides of a triangle?

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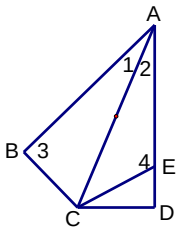

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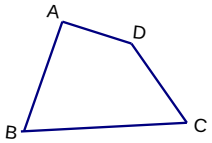

Q 5:

As shown in the figure, $\overline{AC}=\overline{BD}$, $\overline{CE}=\frac{2}{5}\overline{AC}$

Point E is the midpoint of $\overline{BD}$.

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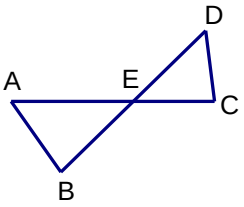

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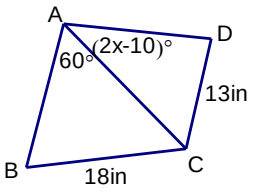

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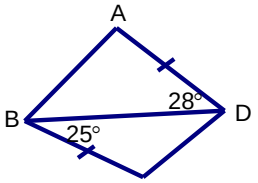

(4) In ΔABC, $\overline{AD}=\overline{BD}$, and $\angle BDC=81^\circ$.

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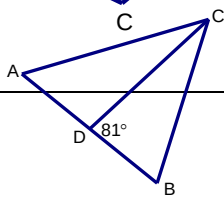

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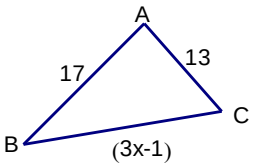

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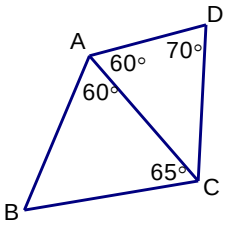

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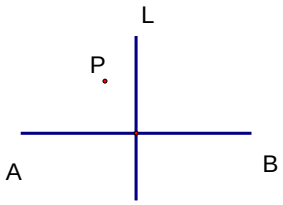

Reference:

https://www.youtube.com/watch?v=ljTVH6UNSUo

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