雙語教學主題(國中八年級下學期教材)三角形邊角關係及樞紐及其逆定理
Topic: THE RELATIONSHIP BETWEEN SIDES AND ANGLES IN TRIANGLES PLUS HINGE THEOREM AND ITS CONVERSE

Vocabulary

CPCTC stands for *corresponding parts of the congruent triangles are congruent*		兩全等三角形的對應邊和對應角都對應相等	
trichotomy law	三一律	converse theorem	逆定理
measure	測量	segment	線段
triangle inequality theorem	三角形邊角關係定理	exterior angle theorem	外角定理
fundamental	基本的	opposite	相對的
true	正確的	false	錯誤的
exterior angle inequality theorem	外角大於任一內對角	converse of the hinge theorem	樞紐逆定理
absolute value	絕對值	the hinge theorem	樞紐定理
isosceles triangle	等腰三角形	base angles	底角
subtraction	減法	substitution	代替

（我個人很喜歡這個CPCTC的表示法，否則要寫很多中文字…）

老師們好，內容儘量呈現在教學時可能用到的英語，供老師們參考。可以自行節錄選取需要的段落或語句使用。也希望老師們建議和指正。祝教學愉快!

We learned a lot about triangle congruence in previous classes. We discuss congruent sides and congruent angles all the time. However, congruent sides and angles are not the only cases we might encounter when looking at two triangles Remember the trichotomy law of numbers we learned in seventh grade? This law tells us the three possible relationships between two given numbers. Therefore, when we talk about the relationships between sides and angles in triangles, there are some unequal relationships, too. Let’s dig into it!

(我們前面幾堂課，學的都是三角形中相等的邊，相等的角，以及三角形的全等。大家還記得七年及時候學過的三一律嗎?
三一律:在任意兩個數 a 和 b 之間，下列三種關係:

$a>b$, $a=b$, 或 $a<b$

恰只會有一種關係成立

在數量中，除了相等，也有數與數之間的不等關係。在三角形中，同樣地，有相等的關係，也有一些不相等，不全等的情形會發生，我們一起來探討吧!)

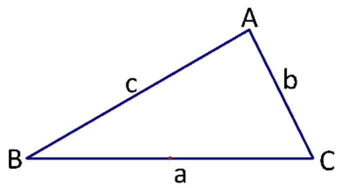
Recall:

Trichotomy law(三一律)
For any two values a and b (a and b are numbers we know, such as real numbers):
Exactly one of the following statements is true:

$a>b$, $a=b$, or $a<b$

We will talk about some fundamental inequality relationships in triangles. The following side-angle relationships only occur in **one** triangle. Let’s go.

Triangle inequality theorem:
In a triangle, the sum of the measure of any two sides is greater than the measure of the third side.(三角形任意兩邊和大於第三邊)



That is:

In ΔABC, let the measure of $\overline{AB}=c$,

the measure of $\overline{BC}=a$, and the measure of $\overline{AC}=b$.

$a+b>c$

$a+c>b$

$b+c>a$

These three statements are always true.
Explain:
It’s easy to understand why they are true.
For instance,

$b+c>a$

We know that the distance between point B and point C is the measure of segment BC. It's also the **shortest** distance from point B to point C. Any other measure of paths from B to C is longer than the measure of segment BC.

That is: the measure of the distance from point B to point A (the length c) and then to point C (the length b) is longer than the measure of segment BC (the length a)

(上面的說明，是用兩點間以直線距離最近做簡單說明，等於沒說。老師們請自行用自己的方式說明，操作，動畫等等。有需要也可以跟我聯絡，一起討論。).

Meanwhile, we can easily get the results below by definition of subtraction.

$a>c-b$

$c>b-a$

$b>a-c$

Because the measure of the side length is always positive, we revise the statements as:

$a+b>c>|a-b|$

(read as" a plus b is greater than c and c is greater than the absolute value of a minus b")

$a+c>b>|a-c|$

$b+c>a>|b-c|$

In a triangle:

A side length is shorter than the sum of the other two side lengths and longer than the absolute value of the difference of the other two side lengths.

(三角形中，任意兩邊和大於第三邊，且任意兩邊差的絕對值小於第三邊)

Please review all the above information if you don't remember them well. It will help you when we learn the other topics in this section

There are some examples for applying the triangle inequality theorem.

Ex 1:

Which of the following could be the sides of a triangle?

A. 1,2,3　　B. 3,5,7　　C. 6,6,6

Sol:

A. 1+2>3 3>3 false

1+3>2 4>2 true

2+3>1 5>1 true so 1,2,3 can' t be the sides of a triangle.

B. 3+5>7 8>7 true

3+7>5 10>5 true

5+7>3 12>3 true so 3,5,7 can be the sides of a triangle.

C. 6+6>6 12>6 true so 6.6.6 can be the sides of a triangle.

So A no, B and C yes

Ex 2:

Two sides of a triangle measure 7cm and 11cm. Write down an inequality that represents the range of values for the possible lengths of the third side.

Sol:

Remember the result that we just learned?

The sum of the two sides > the third side> the difference of the two sides

Let the length of the third side be x, then

$7+11>x>11-7$

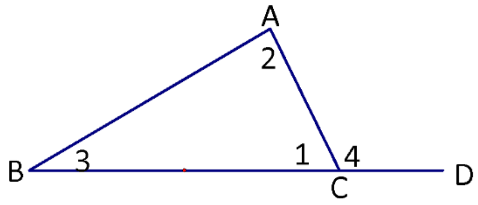
$18>x>4$

That is:

The range of values for the length of the third side is less than 18 and greater than 4.

One more triangle inequality theorem to go—the exterior angle theorem.

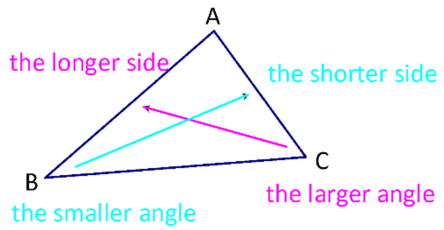
The exterior angle inequality theorem:
An exterior angle of a triangle is greater than any of its non-adjacent angles(remote interior angles).



In the figure on the right,
∠1, ∠2, and ∠3 are interior angles of triangle ABC.
∠4 is the adjacent angle of ∠1.
Then ∠4>∠3 and also ∠4>∠2.
Explain:
We learned the exterior angle theorem before. It says
An exterior angle of a triangle equals the sum of the two remote interior angles.
∠4 is the adjacent angle of ∠1.
So ∠4 is an exterior angle of triangle ABC.
That is: ∠4=∠2+∠3
(by definition of subtraction, ∠2 and ∠3 are positive).

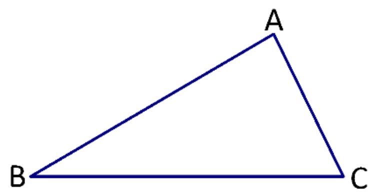
The exterior angle inequality theorem can be applied to the following discussion.
We are going to discuss the side angle relationships in a triangle.
a. The side opposite the larger angle in a triangle is longer. (大角對大邊)

In ΔABC, if $\angle C > \angle B$, then $\overline{AB} > \overline{AC}$



We'll see why it is true now.

In ΔABC, if $\angle C > \angle B$, then $\overline{AB} > \overline{AC}$



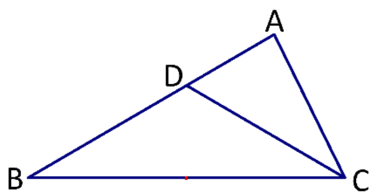
Pf:

(To the students:
The following strategy is what we always use
when we don't have much information on hand. We create useful conditions.
And this is what we usually do:
CONSTRUCT A SMALLER PART FROM A BIGGER PART to meet our needs when given some larger or smaller relations among angles or sides.)

(給學生
當已知條件不足，又提到邊角的大小關係，我們常常使用的策略是
在大的邊或角上擷取一份小的邊或角以供使用。)

Since given $\angle C > \angle B$, we can construct an angle BCD such that $\angle BCD = \angle B$, as shown in Figure 1.

Then $\overline{BD} = \overline{CD}$ …



(In a triangle, sides opposite congruent angles are congruent.)

In ΔACD,

$\overline{AD} + \overline{CD} > \overline{AC}$ (Triangle inequality theorem) Figure 1

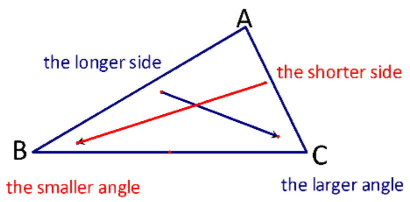
$\Rightarrow$ $\overline{AD} + \overline{BD} > \overline{AC}$ (from)

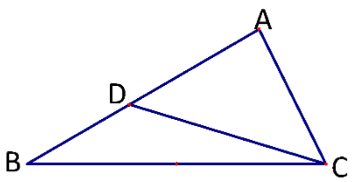
$\Rightarrow$ $\overline{AB} > \overline{AC}$

i.e. If ∠C >∠ B, then $\overline{AB} > \overline{AC}$。

b. The angle opposite the longer. side in a triangle is larger（大邊對大角）

In ΔABC, if $\overline{AB} > \overline{AC}$, then ∠C >∠ B,





Pf:

Since given $\overline{AB} > \overline{AC}$, we choose a point D on $\overline{AB}$ such that

$\overline{AD} = \overline{AC}$, as shown in Figure 1.

Then ΔABC is an isosceles triangle.

∠ACD=∠ADC …

(The base angles in isosceles triangles are congruent.) Figure 1

Now let’ s look at ΔBCD.

∠ADC=∠BCD+∠B (The exterior angle theorem)

⇒ ∠ADC>∠B (The exterior angle inequality theorem)

⇒ ∠ACD=∠ADC>∠B (from)

However,

∠ACB=∠ACD+∠BCD

⇒ ∠ACB>∠ACD (∠BCD>0)

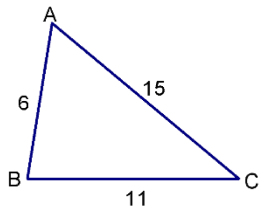
Then we get

∠ACB>∠ACD=∠ADC>∠B

That is:

∠ACB>∠B

i.e. If $\overline{AB} > \overline{AC}$, then ∠C >∠ B。



Ex 3:

Identify the largest angle and the smallest angle in triangle ABC shown on the right.

Sol:

$\overline{AC}$=15 is the longest side

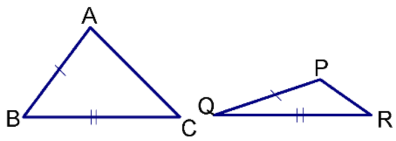
So ∠B opposite the longest side is the largest angle.

$\overline{AB}$=6 is the shortest side

So ∠C opposite the shortest side is the smallest angle.

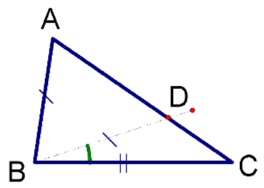
We now discuss another triangle inequality theorem—the hinge theorem and its converse. Unlike other triangle inequality theorems we mentioned, this hinge theorem discusses the inequalities between **two triangles**.

The hinge theorem:
If two sides of one triangle are congruent to two sides of another triangle, and the included angle of the first is larger than the included angle of the second, then the third side of the first triangle is longer than the third side of the second triangle.



In ΔABC and ΔPQR, given $\overline{AB}=\overline{PQ}$ and $\overline{BC}=\overline{QR}$. If $\angle B>\angle Q$, then $\overline{AC}>\overline{PR}$.

Pf:
(The result can be easily seen from the figure above, but we still give formal proof here.)
In Figure 1, construct an angle DBC which is congruent to the angle

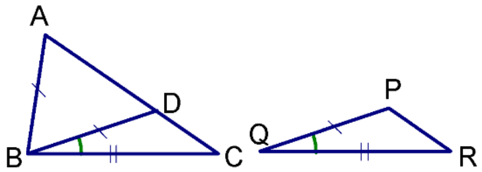


PQR.

and get $\overline{BD}=\overline{QP}$.

Figure 1

1. If point D lies on $\overline{AC}$, as shown in Figure 2.



Then in ΔDBC and ΔPQR,

$\overline{BC} = \overline{QR}$ (Given)

$\angle$DBC=$\angle$PQR and

Figure 2

$\overline{BD} = \overline{QP}$ (Constructed)

Then $\Delta DBC \cong \Delta PQR$ (SAS)

We get $\overline{CD} = \overline{PR}$ (CPCTC)

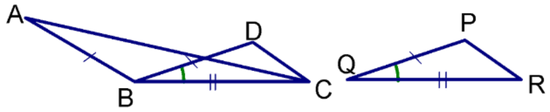
Since $\overline{AC} = \overline{AD} + \overline{CD} = \overline{AD} + \overline{PR}$

$\Rightarrow$ $\overline{AC} > \overline{PR}$ ($\overline{AD} > 0$)

That is: in this case, $\angle B > \angle Q \Rightarrow \overline{AC} > \overline{PR}$.

2. If point D does not lie on $\overline{AC}$, as shown in Figure 3.

Then $\Delta DBC \cong \Delta PQR$ (SAS)



$\overline{CD} = \overline{PR}$ (CPCTC)

(Same as above)

(Nevertheless, we need some information

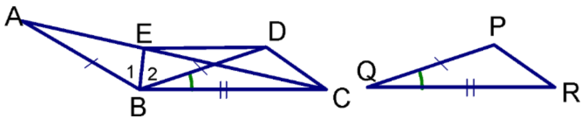
for applying the triangle inequality theorem.

Figure 3

Remember the angle bisector property we learned before?

Construct $\overline{BE}$ as the angle bisector of ∠ABD and intersects $\overline{AC}$ at point E.

Connect $\overline{DE}$. See Figure 4.



in $\triangle$ABE and $\triangle$DBE,

$\overline{AB} = \overline{BD}$ (Given)

∠1=∠2 (Definition of angle bisector) Figure 4

$\overline{BE} = \overline{BE}$ (Reflexive property)

Then $\triangle ABE \cong \triangle DBE$ (SAS)

$\Rightarrow$ $\overline{AE} = \overline{DE}$ (CPCTC)

Look at $\triangle$CDE,

$\overline{CE} + \overline{DE} > \overline{CD}$ (Triangle inequality theorem)

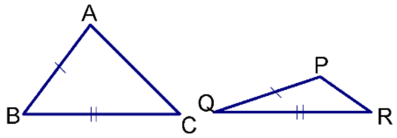
$\Rightarrow$ $\overline{CE} + \overline{AE} > \overline{PR}$ ($\overline{AE} = \overline{DE}$ and $\overline{CD} = \overline{PR}$)

i.e. $\overline{AC} > \overline{PR}$.

This proof is for your reference, the more important thing is we learn how to utilize the Hinge theorem.

The **converse** of hinge theorem:

If two sides of one triangle are congruent to two sides of another triangle, and the third side of the first triangle is longer than the third side of the second, then the included angle of the first is larger than the included angle of the second triangle.



In ΔABC and ΔPQR, given $\overline{AB} = \overline{PQ}$ and $\overline{BC} = \overline{QR}$. If $\overline{AC} > \overline{PR}$, then ∠B>∠Q.

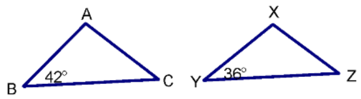
It' s quite obvious, we are not doing the proving here.

Let' s give an example.

Ex 4:

1. Compare the sides and angles and fill in the blanks with a '>' or '<' symbol.

   In ΔABC and ΔXYZ, $\overline{AB} = \overline{XY}$, $\overline{BC} =$



$\overline{YZ}$,

∠B=42°, and ∠Y=36°.

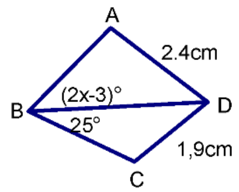
Then $\overline{AC}$ ______ $\overline{XZ}$

Sol:

42°>36°. ∠B=42°, and ∠Y=36°.

⇒ $\overline{AC}$ _>_ $\overline{XZ}$. (the hinge theorem)

2. In $\triangle ABD$ and $\triangle CBD$, $\overline{AB}=\overline{BC}$, $\overline{AD}$=2.4cm, $\overline{CD}$



=1.9cm.

$\angle ABD=(2x-3)^{\circ}$, and $\angle Y=25^{\circ}$.

Find the range of the values of x.

Sol:

2.4cm>1.9cm

i.e. $\overline{AD} > \overline{CD}$

$\Rightarrow \angle ABD > \angle DCB$

$\Rightarrow (2x-3)^{\circ} > 25^{\circ}$ (converse of the hinge theorem)

$\Rightarrow 2x-3>25$

Simplify the inequality

Get $x>14$

Reference:

https://www.youtube.com/watch?v=1jTVH6UNSUo

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