PH Wu

Real Numbers

Class: ______ Number: ______ Name: ____________

1. Rational number

A rational number( ) is any number that you can write in the form $\frac{a}{b}$, where a and b are integers( ) and $b \neq 0$. A rational number in decimal( ) form is either a terminating( ) decimal( ) such as 0.275 or a repeating( ) decimal such as 0.1818…, which you can write as $0.\overline{18}$.

Example 1

Convert the following fractions to decimals. (1) $\frac{5}{8}$ (2) $\frac{2}{7}$.

Solution

Example 2

Convert the following decimals to fractions. (1) 0.12 (2) $1.\overline{34}$ (3) $0.7\overline{6}$.

Solution

2. Irrational numbers and Real numbers

An irrational number(   ) cannot be represented as the quotient(   ) of two integers. In decimal form, irrational numbers do not terminate or repeat.

True or False

(   ) 1. π is an irrational number.

(   ) 2. $\sqrt{4}$, $2\sqrt{5}$, and $3\sqrt{7}$ are all irrational numbers.

3. A farmer has 120 bushels of beans for sale at a farmer's market. He sells an average of $15\frac{3}{4}$ bushels each day. After 6 days, what is the change in the total number of bushels the farmer has for sale at the farmer's market?

Solution

4. Prove: $2+\sqrt{3}$ is an irrational number.

Solution

Real Numbers

Warm up

Hi everyone, please go back to your seats and get the worksheet. Today, let's review what we've covered this week. The topic is real numbers. Please give me some examples of natural numbers, integers, decimals, terminating decimal, repeating decimal, rational numbers, and irrational numbers. Okay, very good. Let's look at part 1 on the worksheet.

Vocabulary

1. rational(有理的) 2. Integer(整數) 3. decimal(小數) 4. terminating(終止的) 5. decimal(小數) 6. repeating(重覆的) 7. fraction(分數) 8. long division(長除法) 9. denominator(分母) 10. recurring period (循環節) 11. irrational(無理的) 12. quotient(商數) 13. approximately(大約地) 14. infinitely(無限地) 15. proof by contradiction(反證法)

Illustrations

Part 1. (Student's name), please read point 1 for us.

(After reading)

In short, a rational number is a fraction, and the denominator cannot be zero. Another form of a rational number is a decimal. It can be a terminating decimal, like 0.275, which stops at a certain point, or a repeating decimal, like 0.181818…, which continues infinitely. Let's see example 1. Convert the **fractions**[7] to decimals.

Example 1

(1) The first question is converting 5 over 8 to a decimal number. We can use **long division**[8] to get the answer.

$$
\begin{array}{rcccc}
 & 0. & 6 & 2 & 5 \\ \cline{2-5}
8 & \multicolumn{1}{|c}{5} & 0 & & \\
 & 4 & 8 & & \\ \cline{2-3}
 & & 2 & 0 & \\
 & & 1 & 6 & \\ \cline{3-4}
 & & & 4 & 0 \\
 & & & 4 & 0 \\ \cline{4-5}
 & & & & 0
\end{array}
$$

Then, we have $\frac{5}{8} = 0.625$.

Alternatively, we also can use the expansion of a fraction. By converting the **denominator**[9] to a power of 10, then we get

$$\frac{5}{8}=\frac{5}{2^3}=\frac{5\times 5^3}{2^3\times 5^3}=\frac{625}{10^3}=0.625.$$

(2) The second question is converting 2 over 7 to a decimal number. Using long division to divide 2 by 7, we have

$$
\begin{array}{r|cccccccccc}
 & 0. & 2 & 8 & 5 & 7 & 1 & 4 & 2 & 8 & \dots \\ \hline
7 & 2 & 0 \\
 & 1 & 4 \\ \cline{2-3}
 & & 6 & 0 \\
 & & 5 & 6 \\ \cline{3-4}
 & & & 4 & 0 \\
 & & & 3 & 5 \\ \cline{4-5}
 & & & & 5 & 0 \\
 & & & & 4 & 9 \\ \cline{5-6}
 & & & & & 1 & 0 \\
 & & & & & & 7 \\ \cline{6-7}
 & & & & & & 3 & 0 \\
 & & & & & & 2 & 8 \\ \cline{7-8}
 & & & & & & & 2 & 0 \\
 & & & & & & & 1 & 4 \\ \cline{8-9}
 & & & & & & & & 6 & 0 \\
 & & & & & & & & 5 & 6 \\ \cline{9-10}
 & & & & & & & & & \dots
\end{array}
$$

Hence, we have $\frac{2}{7}=0.28571428\ldots=0.\overline{285714}$ (2 over 7 equals 0 point 2 8 5 7 1 4 2 8 dat dat dat equals 0 point 2 8 5 7 1 4 repeating.)

Example 2

(1) For the first question, we know that 0.12 is equal to 12 over 100.

So, $0.12=\frac{12}{100}=\frac{3}{25}$.

(2) To get rid of the repeating digits, we can multiply 10 to the power of its **recurring period**[10]. For example, $1.\overline{34}$, the repeating digits "34" has a recurring period of 2, so we can multiply $1.\overline{34}$ by 10^2, namely $1.\overline{34}\times 10^2=134.\overline{34}$.

If $x=1.\overline{34}$, then $100x=134.\overline{34}$

Subtracting two equations, $100x-x=134.\overline{34}-1.\overline{34}$, leads to $99x=133$.

Hence, $x = \frac{133}{99}$.

(3) Since $0.7\overline{6}$, the repeating digit "6" has a recurring period of 1, then we multiply $0.7\overline{6}$ by 10^1,

namely $0.7\overline{6} \times 10^1 = 7.6\overline{6}$.

If $x = 0.7\overline{6}$, then $10x = 7.6\overline{6}$

Subtracting two equations, $10x - x = 7.6\overline{6} - 0.7\overline{6}$, leads to $9x = 6.9$.

Hence, $x = \frac{6.9}{9} = \frac{69}{90} = \frac{23}{30}$.

Part 2. (Student's name), please read point 2 for us.

(After reading)

Let's move on to True/False questions.

(1) Is π an irrational number? Since π is **approximately**[13] 3.141592653589793238... and continues **infinitely**[14] without repeating, π is an irrational number. Hence, it is a true statement.

(2) Are $\sqrt{4}$, $2\sqrt{5}$, and $3\sqrt{7}$ all irrational numbers? Since $\sqrt{4}$ can be simplified to 2, which is a rational number, the answer to this question should be false.

Part 3. (Student's name), please read point 3 for us.

(After reading)

To find the change in the total number of bushels the farmer has for sale after 6 days, we can use the given average daily sales and calculate accordingly.

"Total bushels sold in 6 days" is "Average bushels sold per day × Number of days." Hence, we have $15\frac{3}{4}$ multiplied by 6, leads to $15\frac{3}{4} \times 6 = \frac{63}{4} \times 6 = \frac{189}{2}$. And, the change in total bushels equals $120 - \frac{189}{2} = \frac{105}{2} = 52.5$.

Therefore, after 6 days, the change in the total number of bushels the farmer has for sale at the farmer's market is 52.5 bushels.

Part 4. To prove that $2 + \sqrt{3}$ is an irrational number, we can use a **proof by contradiction**[15].

Suppose $2 + \sqrt{3}$ is a rational number.

Then there exist integers *a* and *b* ($b \neq 0$) such that $2+\sqrt{3}=\frac{a}{b}$. Isolating $\sqrt{3}$, we have $\sqrt{3}=\frac{a}{b}-2$. Since *a*, *b* and 2 are all integers, $\frac{a}{b}-2$ is a rational number minus an integer, which is also a rational number. We know that $\sqrt{3}$ is an irrational number. However, the assumption led to conclude that $\sqrt{3}$ is rational, which is a contradiction. Hence, the initial assumption that $2+\sqrt{3}$ is rational must be false. Therefore, $2+\sqrt{3}$ is an irrational number.

References

1. 許志農、黃森山、陳清風、廖森游、董涵冬（2019）。*數學 1：單元 1 實數*。龍騰文化。
2. Randall I. Charles, Basia Hall, Dan Kennedy, Allan E. Bellman, Sadie Chavis Bragg, William G. Handlin, Stuart J. Murphy, & Grant (2012). *Pearson Algebra 2 Common Core.* Pearson.