雙語教學主題(國中八年級下學期教材):三角形全等
Practice/test: Applications of triangle congruenc

老師們好，這是全等三角形的應用練習。這份教材的前半是題目加解答，後半是題目，方便老師們參考使用。老師們可以依據學生的需求選取適當的題目供學生做小測或是練習。

Vocabulary

<table>
<tr><td colspan="2">CPCTC
stands for corresponding parts of the congruent triangles are congruent</td><td colspan="2">兩全等三角形的對應邊和對應角都對應相等</td></tr>
<tr><td colspan="2">Triangle congruence theorem</td><td colspan="2">全等三角形全等性質(定理)</td></tr>
<tr><td>憑</td><td>對應邊</td><td>corresponding angles</td><td>對應角</td></tr>
<tr><td>congruent sides</td><td>等邊</td><td>Congruent angles</td><td>等角</td></tr>
<tr><td>included side</td><td>夾邊</td><td>included angle</td><td>夾角</td></tr>
<tr><td>bisect</td><td>平分</td><td>statement</td><td>敘述、語句</td></tr>
<tr><td>TWO COLUMN PROOFS</td><td>兩行式證明</td><td>Reflexive property</td><td>自反性、反身性(自已等於自己)</td></tr>
<tr><td>bisect</td><td>平分</td><td>angle bisector</td><td>角平分線</td></tr>
<tr><td>fill in the blanks</td><td>填空</td><td>trisect</td><td>三等分</td></tr>
<tr><td>Quadrilateral</td><td>四邊形</td><td>midpoint</td><td>中點</td></tr>
<tr><td>Triangle angle sum theorem</td><td>三角形內角和定理</td><td>intersect</td><td>相交於</td></tr>
<tr><td>equilateral triangle</td><td>正三角形</td><td>collinear</td><td>共線</td></tr>
<tr><td>vertical angle</td><td>對頂角</td><td></td><td></td></tr>
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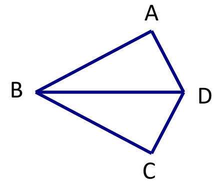
(我個人很喜歡這個 CPCTC 的表示法，否則要寫很多中文字...)

Q 1:

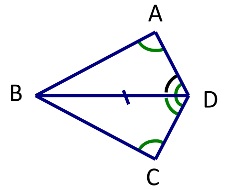


In $\triangle ABD$ and $\triangle CBD$, $\angle A = \angle C$, $\overline{BD}$ bisects $\angle ADC$.

Prove that: $\triangle ABD \cong \triangle CBD$

老師們，這個例子是讓大家參考，在國外，很多的三角形證明問題，會用以下的形式呈現。所謂的 TWO COLUMN PROOFS. 我們當然還是沿用我們習慣的形式就好。

ANSWER:



Pf:

Let’s mark the congruent sides and angles on the triangles.

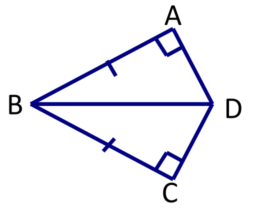
	statement	reasons
In $\triangle ABD$ and $\triangle CBD$,		
	$\angle A \cong \angle C$	Given
	$\overline{BD}$ bisects $\angle ADC$	Given
	$\angle ADB \cong \angle CDB$	Definition of angle bisector
We have two pairs of congruent angles, we need a pair of congruent sides, so we use	$\overline{BD} \cong \overline{BD}$	Reflexive property
Then we get	$\triangle ABD \cong \triangle CBD$	AAS

其實跟我們平常的證明過程大同小異，重要的條件和理由都有說明就好。不過，我們的邊角相等用”=”，外國人會用”≅”符號；現在他們也沒有那麼嚴格要求了。

Q 2:

Please fill in the blanks according to the information shown in the figure.

In $\triangle ABD$ and $\triangle CBD$,



$\overline{AB} =$ ________ (Given)

$\angle BAD =$ ________ $= 90^\circ$ (Given)

$\overline{BD} =$ ________ (Reflexive property)

So $\triangle ABD \cong \triangle CBD$ (________)

ANSWER:

In $\triangle ABD$ and $\triangle CBD$,

$\overline{AB} =$ ____$\overline{CB}$______ (Given)

$\angle BAD =$ __$\angle BCD$________ $= 90^\circ$ (Given)

$\overline{BD} =$ ___$\overline{BD}$____ (Reflexive property)

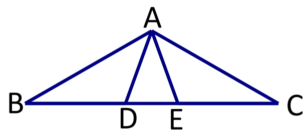
So $\triangle ABD \cong \triangle CBD$ (___RHS_)

Q 3:

In isosceles triangle ABC, $\overline{AB} = \overline{AC}$,

$\overline{AD}$ and $\overline{AE}$ trisect $\angle BAC$.

Prove that $\triangle ABE \cong \triangle ACD$

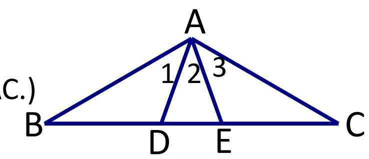


ANSWER:

Pf:

(1) Let $\angle BAD = \angle 1$, $\angle DAE = \angle 2$, and $\angle CAE = \angle 3$.

Then $\angle 1 = \angle 2 = \angle 3$ ($\overline{AD}$ and $\overline{AE}$ trisect $\angle BAC$.)



In $\triangle ABE$ and $\triangle ACD$,

$\overline{AB} = \overline{AC}$ (Given)

$\angle BAE = \angle 1 + \angle 2$

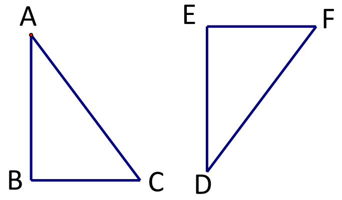
$= \angle 2 + \angle 3$

$= \angle CAD$

And $\angle B = \angle C$ (In a triangle, the angles opposite equal sides are congruent)

Then $\triangle ABE \cong \triangle ACD$ (ASA)

Q 4:



In $\triangle ABC$ and $\triangle DEF$, $\overline{AB} \perp \overline{BC}$, $\overline{DE} \perp \overline{EF}$,

$\overline{AC} = \overline{DF}$, and $\overline{BC} = \overline{EF}$. If $\angle A = (2X+1)^\circ$, $\angle F = (3X-6)^\circ$.

Find (1) The value of x

(2) The measure of $\angle D$

ANSWER:

Sol:

Mark the given features on the figure.

(1) We can easily see that in $\triangle ABC$ and $\triangle DEF$,

$\overline{AC} = \overline{DF}$ (Given)

$\overline{BC} = \overline{EF}$ (Given)

And $\angle B = \angle E = 90^\circ$ ($\overline{AB} \perp \overline{BC}$, $\overline{DE} \perp \overline{EF}$)

$\triangle ABC \cong \triangle DEF$ (RHS)

Then $\angle A = \angle D$, $\angle C = \angle F$ (CPCTC)

$\angle A = (2X+1)^\circ$, $\angle C = \angle F = (3X-6)^\circ$

In $\triangle ABC$,

$\angle A + \angle B + \angle C = 180^\circ$ (Triangle angle sum theorem)

$(2X+1)^\circ + 90^\circ + (3X-6)^\circ = 180^\circ$

$5x - 5 = 90$ (Combine like terms)

$5x = 95$

$x = 19$ #

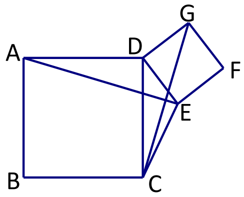
(2) $\angle D = \angle A = (2X+1)^\circ = (2 \bullet 19 + 1)^\circ$ (Replace x with 19)

$= 39^\circ$ #

Q 5:

Quadrilateral ABCD and DEFG are squares.

Prove that $\Delta ADE \cong \Delta CDG$.

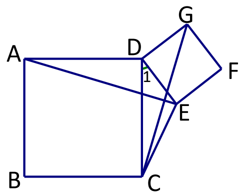


ANSWER:

pf:

Since quadrilateral ABCD and DEFG are squares, we know that four sides of a square are equal and four interior angles are right angles.

label $\angle CDE$ as $\angle 1$ shown in the figure on the right.



In ΔADE and ΔCDG,

we see that $\overline{AD}=\overline{CD}$, $\overline{DE}=\overline{DG}$ (Both are congruent sides of square ABCD and square DEFG respectively.)

Also $\angle ADC=\angle EDG=90^\circ$ (Both are right angles of square ABCD and square DEFG)

So we get

$$\begin{aligned}\angle ADE &= \angle ADC+\angle 1\\ &=90^\circ+\angle 1\\ &=\angle EDG+\angle 1\\ &=\angle CDG\end{aligned}$$

Therefore,

$\Delta ADE \cong \Delta CDG$ (SAS)

Q 6:

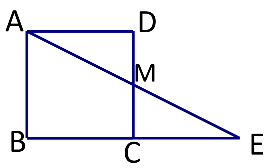
Quadrilateral ABCD is a square.

Point M is the midpoint of $\overline{CD}$. Let line AM and line BC intersect each other at point E.

(1) Prove that $\Delta ADM \cong \Delta ECM$

(2) If $\overline{CE}=5$, find the area of ΔABE

(3) Find the length of $\overline{ME}$



ANSWER:

Sol:

(1) Pf:

In ΔADM and ΔECM,

$\angle ADM = \angle BCM = 90^\circ$ (The angle measurement of any interior angle of a square is 90°)

And

$\angle ECM = 180^\circ - \angle BCM$ (Line BE forms a straight angle)

$= 180^\circ - 90^\circ$

$= 90^\circ$

$\angle ADM = \angle ECM = 90^\circ$

$\overline{CM} = \overline{DM}$ (Point M is the midpoint of $\overline{CD}$)

We need one more congruent sides or congruent angles, so we see

$\angle AMD = \angle CME$ (vertical angles are congruent)

Then

$\Delta ADM \cong \Delta ECM$ (ASA)

(2) $\Delta ADM \cong \Delta ECM$ (From (1))

$\overline{AD} = \overline{CE}$ (CPCTC)

So $\overline{AD} = \overline{CE} = 5$ (Given)

Then $\overline{AB} = \overline{BC} = \overline{CD} = \overline{AD} = 5$ (All the sides are congruent in a square)

The area of $\Delta ABE = \frac{1}{2}\overline{AB} \cdot \overline{BE}$

$= \frac{1}{2}\overline{AB} \cdot \left(\overline{BC} + \overline{CE}\right)$

$= \frac{1}{2} 5 \cdot (5+5)$

$= 25_{\#}$

(3) Let's look right into ΔECM

$\angle ECM = 90^\circ$

$\overline{CM} = \overline{DM}$ (From (1))

So $\overline{CM} = \overline{DM} = \frac{1}{2}\overline{CD} = \frac{5}{2}$

Since ΔECM is a right triangle,

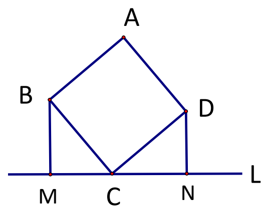
$\overline{ME}^2 = \overline{MC}^2 + \overline{CE}^2$ (The Pythagorean Theorem)

$= \left(\frac{5}{2}\right)^2 + 5^2$

$= \frac{5 \cdot 25}{4}$

$\overline{ME} = \frac{5\sqrt{5}}{2}{}_{\#}$

Q 7:



Quadrilateral ABCD is a square. Point C is on line L.

$\overline{BM} \perp L$ and intersects line L at point M,

$\overline{DN} \perp L$ and intersects line L at point N.

Then (1) Prove that $\Delta BCM \cong \Delta CDN$

(2) If $\overline{AB}=10$, $\overline{DN}=6$, find the length of $\overline{MN}$

ANSWER:

(1) pf:

Let $\angle CBM=\angle 1$, $\angle BCM=\angle 2$, and $\angle DCN=\angle 3$ as shown in the figure on the right.

$\angle BCD=90^\circ$ (All interior angles of a square are 90°)

In ΔBCM, $\angle 1+\angle 2+90^\circ=180^\circ$

Get $\angle 1+\angle 2=90^\circ$

And $\angle 3+\angle 2+\angle BCD=180^\circ$ ($\angle 3$, $\angle 2$, and $\angle BCD$ form a straight angle)

Get $\angle 3+\angle 2=90^\circ$ ($\angle BCD=90^\circ$)

$\angle 1+\angle 2=90^\circ=\angle 3+\angle 2$

So $\angle 1=\angle 3$

In ΔBCM and ΔCDN,

$\angle BMC=\angle CND=90^\circ$ ($\overline{BM} \perp L$, $\overline{DN} \perp L$ Definition of perpendicular line)

$\angle 1=\angle 3$

$\overline{BC}=\overline{CD}$ (All sides of a square are congruent)

Then $\Delta BCM \cong \Delta CDN$ (AAS)

(2) in ΔCDN,

$\overline{DN}=6$ (Given)

$\overline{CD}=\overline{AB}=10$, (All sides are congruent in a square)

Then $\overline{CN}^2=\overline{CD}^2-\overline{DN}^2$ (The Pythagorean Theorem)

$=10^2-6^2$

$=64$

$\overline{CN}=8$

And $\overline{MC}=\overline{DN}=6$ (From (1), CPCTC)

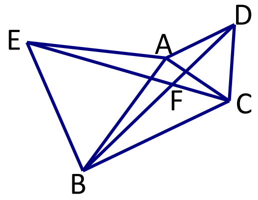
Therefore,

$\overline{MN} = \overline{MC} + \overline{CN}$

=6+8

-14#

Q 8:



ΔABE and ΔACD are equilateral triangles. $\overline{BD}$ and $\overline{CE}$ intersect each other at point F.

(1) Prove that $\Delta ACE \cong \Delta ADB$.

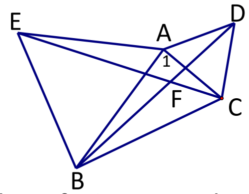
(2) Find the measure of $\angle DFE$

ANSWER:

pf:

(1) Since ΔABE and ΔACD are equilateral triangles, we know that three sides of an equilateral triangle are congruent and three interior angles are equal to 60°.

Label $\angle BAC$ as $\angle 1$ shown in the figure on the right.



In ΔACE and ΔADB,

we see that $\overline{AC} = \overline{AD}$, $\overline{AE} = \overline{AB}$ (Both are congruent sides of ΔABE and ΔACD respectively.)

Also $\angle BAE = \angle CAD = 60^\circ$ (Both are interior angles of equilateral triangles.)

We get $\angle EAC = \angle EAB + \angle 1$

$= 60^\circ + \angle 1$

$= \angle CAD + \angle 1$

$= \angle BAD$

Therefore,

$\Delta ACE \cong \Delta ADB$ (SAS)

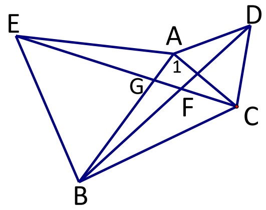
(2) $\overline{AB}$ and $\overline{CE}$ intersect each other at point G as shown in the figure on the right.

Since we look for the measure of an angle,

we need to get the connection to what we have here

--- The interior angles of an equilateral triangle are all equal to 60°.

So let's look into ΔAEG and ΔFBG,



In ΔAEG and ΔFBG,

$\angle AEG = \angle FBG$ (From (1), $\Delta ACE \cong \Delta ADB$, CPCTC)

$\angle AGE = \angle FGB$ (Congruent vertical angles)

Then $\angle BFG=180°-\angle FGB-\angle FBG$

$=180°-\angle AGE-\angle AEG$ (Triangle angle sum theorem)

$=\angle EAG$

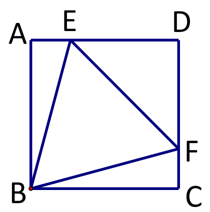
$=60°$

$\angle DFE=180°-\angle BFG$ (Points B, F, and D are collinear)

$=180°-60°$

$=120°$ #

Q 9:



In square ABCD, $\overline{DE}=\overline{DF}$, $\angle ABE=20°$.

(1) Prove that $\Delta ABE \cong \Delta CBF$

(2) The measure of $\angle BFE$

ANSWER:

Pf:

(1) ΔABE and ΔCBF,

$\overline{DE}=\overline{DF}$ (Given)

$\overline{AE}\ \overline{AD}-\overline{DE}$

$=\overline{CD}-\overline{DF}$ ($\overline{AD}=\overline{CD}$, all sides of a square are congruent)

$=\overline{CF}$

And $\overline{AB}=\overline{BC}$ (All sides of a square are congruent)

$\angle A=\angle D=90°$ (All interior angles in a square are right angles.)

Then $\Delta ABE \cong \Delta CBF$ (SAS)

(2) From (1), we have

$\angle ABE=\angle CBF$

And $\overline{BE}=\overline{BF}$ (CPCTC)

We need to find the measure of $\angle BFE$, then we look into ΔBFE.

In ΔBFE,

$\angle EBF=\angle ABC-\angle ABF-\angle CBF$

$=90°-20°-20°$

$=50°$

Since $\overline{BE}=\overline{BF}$, ΔBFE is an isosceles triangle.

We get

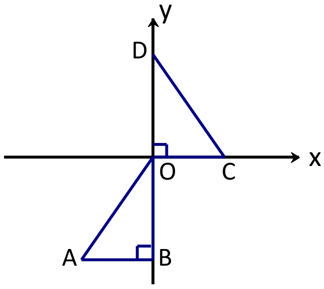
$\angle BEF-\angle BFE$ (The two angles of an isosceles triangle opposite of the two congruent sides are congruent.)

So $\angle BFE=\frac{1}{2}(180°-\angle EBF)$

$= \frac{1}{2}(180° - \angle 50°)$

$= 65°$ #

Q 10:



On the coordinate plane, point O is the origin. Point B and point D are on the y-axis.

In two right triangles ΔABO and ΔCOD,

$\angle ABO = \angle COD = 90°$, the coordinates of point C are (5,0),

$\overline{AO} = \overline{CD} = 13$ and $\angle AOB = \angle CDO$. .

Find the coordinates of point A.

ANSWER:

Sol:

If we want to get the coordinates of point A, we need to know the length of $\overline{AB}$ and $\overline{BO}$. We see that a lot of congruent parts given between ΔABO and ΔCOD. Let's see what we can get here.

In ΔABO and ΔCOD,

$\angle ABO = \angle COD = 90°$

$\overline{AO} = \overline{CD} = 13$

$\angle AOB = \angle CDO$ (Given)

Then $\Delta ABO \cong \Delta COD$ (AAS)

$\overline{AB} = \overline{OC}$

$\overline{BO} = \overline{DO}$ (CPCTC)

Since $\overline{CD} = 13$ and $\overline{OC} = 5$,

$\overline{DO}^2 = \overline{CD}^2 - \overline{OC}^2$

$= 13^2 - 5^2$

$= 144$

We get $\overline{DO} = 12$

Therefore,

$\overline{AB} = \overline{OC} = 5$ and $\overline{BO} = \overline{DO} = 12$

Point A is in the third quadrant,

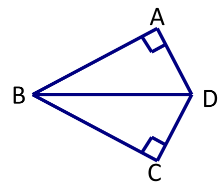
So the coordinates of point A are (-5,-12)

QUESTIONS only:

Q 1:

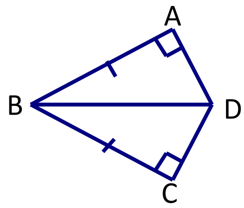
In $\triangle ABD$ and $\triangle CBD$, $\angle A = \angle C = 90^\circ$, $\overline{BD}$ bisects $\angle ADC$.

Prove: $\triangle ABD \cong \triangle CBD$



Q 2:

Please fill in the blanks according to the information shown in the figure.



In $\triangle ABD$ and $\triangle CBD$,

$\overline{AB} =$ ________ (Given)

$\angle BAD =$ ________ $= 90^\circ$ (Given)

$\overline{BD} =$ ________ (Reflexive property)

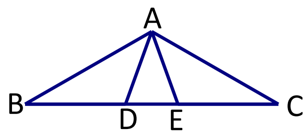
So $\triangle ABD \cong \triangle CBD$ (________)

Q 3:

In isosceles triangle ABC, $\overline{AB} = \overline{AC}$,

$\overline{AD}$ and $\overline{AE}$ trisect $\angle BAC$.

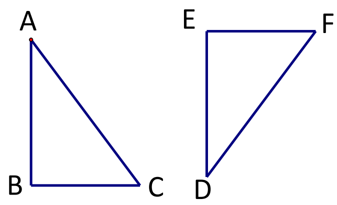
Prove that $\triangle ABE \cong \triangle ACD$



Q 4:

In $\triangle ABC$ and $\triangle DEF$, we have $\overline{AB} \perp \overline{BC}$, $\overline{DE} \perp \overline{EF}$,

and $\overline{AC} = \overline{DF}$, $\overline{BC} = \overline{EF}$. If $\angle A = (2X+1)^\circ$, $\angle F = (3X-6)^\circ$,



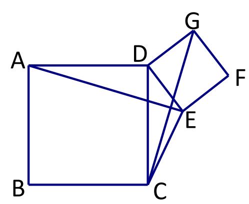
find (1) The value of x

(2) The measure of $\angle D$

Q 5:

Quadrilateral ABCD and DEFG are squares.

Prove that $\triangle ADE \cong \triangle CDG$.



Q 6:

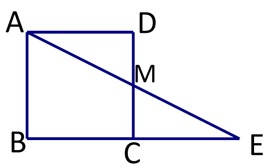
Quadrilateral ABCD is a square.

Point M is the midpoint of $\overline{CD}$. Let line AM and line BC intersect each other at point E.

(1) Prove that $\triangle ADM \cong \triangle ECM$

(2) If $\overline{CE} = 5$, find the area of $\triangle ABE$

(3) Find the measure of $\overline{ME}$



Q 7:

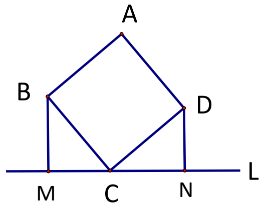
ABCD is a square. Point C is on line L.

$\overline{BM} \perp L$ and intersects line L at point M,

$\overline{DN} \perp L$ and intersects line L at point N.

Then (1) Prove that $\Delta BCM \cong \Delta CDN$

(2) If $\overline{AB}=10$, $\overline{DN}=6$, find the length of $\overline{MN}$

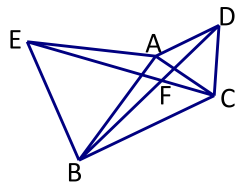


Q 8:

ΔABE and ΔACD are equilateral triangles. $\overline{BD}$ and $\overline{CE}$ intersect each other at point F.

(1) Prove that $\Delta ACE \cong \Delta ADB$.

(2) Find the measure of $\angle DFE$

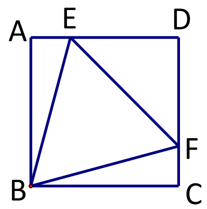


Q 9:

In square ABCD, $\overline{DE}=\overline{DF}$, $\angle ABE=20°$.

(1) Prove that $\Delta ABE \cong \Delta CBF$

(2) The measure of $\angle BFE$



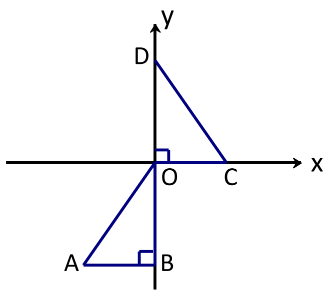
Q 10:

On the coordinate plane, point O is the origin. Point B and point D are on the y-axis.

In two right triangles ΔABO and ΔCOD,

$\angle ABO = \angle COD = 90°$, the coordinates of point C are (5,0), $\overline{AO}=\overline{CD}=13$ and $\angle AOB = \angle CDO$. .

Find the coordinates of point A.



Reference:

教育部國民中學數學 108 課綱

教育部審定國民中學數學科南一、康軒以翰林及第五冊課本

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