雙語教學主題(國中八年級下學期教材):三角形全等

Topic: Triangle congruence theorems

Vocabulary

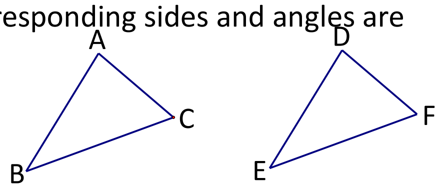
CPCTC stands for corresponding parts of the congruent triangles are congruent		兩全等三角形的對應邊和對應角都對應相等	
Triangle congruence theorem		全等三角形全等性質(定理)	
corresponding sides	對應邊	corresponding angles	對應角
congruent sides	等邊	Congruent angles	等角
included side	夾邊	included angle	夾角

(我個人很喜歡這個 CPCTC 的表示法，否則要寫很多中文字…)

老師們好，內容儘量呈現在教學時可能用到的英語，供老師們參考。可以自行節錄選取需要的段落或語句使用。也希望老師們建議和指正。祝教學愉快!

We have learned that when two figures on a plane are overlapped perfectly, these two figures are congruent. That is when triangle ABC is translated, reflected, or rotated to triangle DEF and they are overlapped completely, these two triangles are congruent. It’s noted as $\Delta ABC \cong \Delta DEF$.

$\Delta ABC \cong \Delta DEF$ means all the corresponding parts from both triangles are congruent. From the figure shown, all the corresponding sides and angles are congruent.



That is:

$\overline{AB} = \overline{DE}$ (corresponding sides)

$\overline{BC} = \overline{EF}$ (corresponding sides)

$\overline{AC} = \overline{DF}$ (corresponding sides)

(segment AB is congruent to segment DE…)

And,

$\angle A = \angle D$ (corresponding angles)

$\angle B = \angle E$ (corresponding angles)

$\angle C = \angle F$ (corresponding angles)

(angle A is congruent to angle D…)

Pay extra attention to the order of letters of the vertices from both triangles as you write down the triangle congruency.

Vertex A corresponds to vertex D, vertex B corresponds to vertex E, and vertex C corresponds to vertex F.

There are five triangle congruence theorems to help us to show if two triangles are congruent.

They are the SSS triangle congruence theorem

the SAS triangle congruence theorem

the ASA triangle congruence theorem

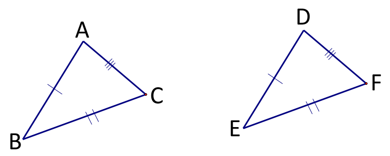
the AAS triangle congruence theorem

the RHS triangle congruence theorem

S stands for side and A stands for angle.

SSS triangle congruence theorem:

If three sides of a triangle are congruent to the corresponding sides of another triangle, these two triangles are congruent.



Given ΔABC and ΔDEF

If $\overline{AB} = \overline{DE}$

$\overline{BC} = \overline{EF}$

$\overline{AC} = \overline{DF}$

Then

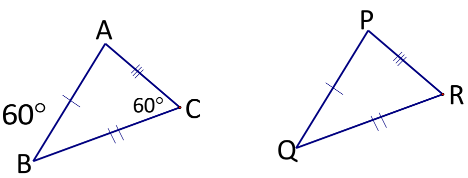
$\Delta ABC \cong \Delta DEF$ (triangle ABC is congruent to triangle DEF)

Let's have an example here.

Ex 1:

Given ΔABC and ΔPQR,

If $\overline{AB} = \overline{PQ}$, $\overline{BC} = \overline{QR}$, $\overline{AC} = \overline{PR}$, and $\angle C = 60°$



Then $\angle R = ?$

Sol:

In ΔABC and ΔPQR,

$\overline{AB} = \overline{PQ}$, $\overline{BC} = \overline{QR}$, $\overline{AC} = \overline{PR}$ (given)

So $\Delta ABC \cong \Delta PQR$ (SSS)

$\Rightarrow \angle R = \angle C = 60°$ # (CPCTC)

The following triangle congruence theorem is SAS.

SAS triangle congruence theorem:

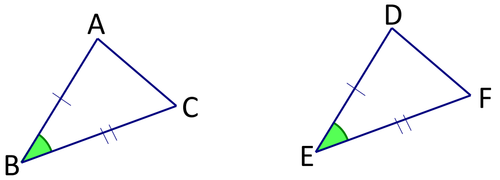
If two sides of a triangle and the included angle are congruent to the corresponding parts of another triangle, these two triangles are congruent.

Given ΔABC and ΔDEF

If $\overline{AB} = \overline{DE}$

$\overline{BC} = \overline{EF}$

$\angle B = \angle E$



Then

$\Delta ABC \cong \Delta DEF$ (triangle ABC is congruent to triangle DEF)

There is an example of the theorem above.

Ex 2:

ΔAOB and ΔCOD are shown in Figure 1.

$\overline{AO} = \overline{CO}$, $\overline{BO} = \overline{DO}$. If $\angle C = 110^\circ$, find the measure of $\angle A$.

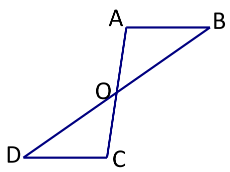


Figure 1

Sol:

In ΔAOB and ΔCOD,

We have

$\overline{AO} = \overline{CO}$, $\overline{BO} = \overline{DO}$

Let's label the congruent sides with dashes in Figure 2.

Look at Figure 2, does it remind you of the relationship between $\angle AOB$ and $\angle COD$?

YES! They are vertical angles. And we learned that vertical angles are congruent.

So $\angle AOB = \angle COD$

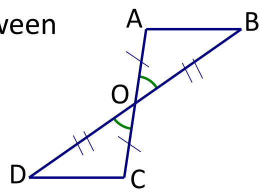


Figure 2

Therefore, In ΔAOB and ΔCOD,

$\overline{AO} = \overline{CO}$, $\overline{BO} = \overline{DO}$ (given)

And $\angle AOB = \angle COD$ (congruent vertical angles)

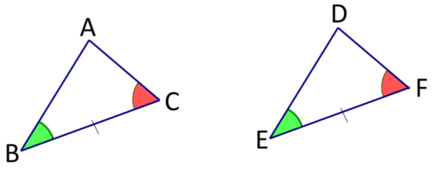
Then $\Delta AOB \cong \Delta COD$ (SAS)

So

$\angle A = \angle C = 110^\circ$ $\angle C = 110^\circ$ # (CPCTC)

Next, we are going to talk about the ASA triangle congruence theorem

ASA triangle congruence theorem:
If two angles of a triangle and the included side are congruent to the corresponding parts of another triangle, these two triangles are congruent.



Given Δ ABC and Δ DEF

If $\overline{BC} = \overline{EF}$

$\angle B = \angle E$

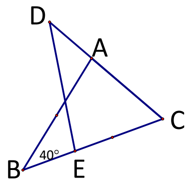
$\angle C = \angle F$

Then

$\Delta ABC \cong \Delta DEF$ (triangle ABC is congruent to triangle DEF)

Let's look at an example.

Ex 3:



Given $\overline{CA} = \overline{CE}$, $\angle BAC = \angle DEC$, and $\angle B = 40°$.

Find the measure of $\angle$D.

Sol:

Method 1:

In Δ ABC and Δ EDC,

$\overline{CA} = \overline{CE}$ (given)

$\angle BAC = \angle DEC$ (given)

$\angle C = \angle C$ (reflexive)

Then $\Delta ABC \cong \Delta EDC$ (ASA)

$\angle D = \angle B = 40°$ # (CPCTC)

Method 2:

In Δ ABC and Δ EDC,

$\angle BAC = \angle DEC$ (given)

$\angle C = \angle C$ (reflexive)

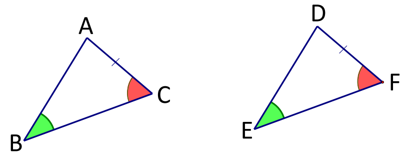
Then

$\angle D = \angle B = 40°$ # (the third angle's theorem)

Let's talk about the AAS triangle congruence theorem which is similar to the ASA triangle congruence theorem.

AAS triangle congruence theorem:

if two consecutive angles of a triangle and the side in a row clockwise or counterclockwise are congruent to the corresponding parts of another triangle, these triangles are congruent.



Given Δ ABC and Δ DEF,

If $\overline{AC} = \overline{DF}$

$\angle B = \angle E$

$\angle C = \angle F$

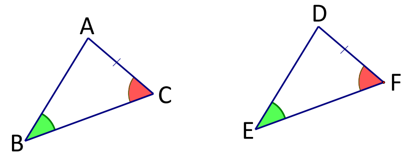
Then

$\Delta ABC \cong \Delta DEF$ (triangle ABC is congruent to triangle DEF)

The ASA triangle congruence theorem and the AAS triangle congruence theorem are equivalent. Why? Let me explain.

The third angle's theorem shows that if two angles of a triangle are congruent to two angles of another triangle, then the third angle of each triangle is congruent.

Let's see:



In Δ ABC and Δ DEF,

$\angle B = \angle E$ and $\angle C = \angle F$

Then the third angle of Δ ABC must be congruent to the third angle of Δ DEF.

That is: $\angle A = \angle D$

We can also do the reasoning:

Since the sum of the interior angles of a triangle is 180°.

Then $\angle A = 180^\circ - \angle B - \angle C$

$= 180^\circ - \angle E - \angle F$

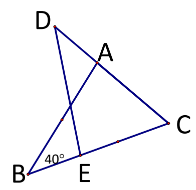
$= \angle D$

That's why I say these two triangle congruence theorems are equivalent. It's more convenient and direct for us to do the reasoning with certain conditions given by the questions.

Let's change the conditions of example 3 above.

Ex 4:

Given $\overline{BC} = \overline{DC}$, $\angle BAC = \angle DEC$, and $\angle B = 40°$.



Find the measure of $\angle D$.

Sol:

Method 1:

In ΔABC and ΔEDC,

$\overline{BC} = \overline{DC}$ (given)

$\angle BAC = \angle DEC$ (given)

$\angle C = \angle C$ (reflexive)

Then $\Delta ABC \cong \Delta EDC$ (AAS)

$\angle D = \angle B = 40°$ # (CPCTC)

Method 2:

In ΔABC and ΔEDC,

$\angle BAC = \angle DEC$ (given)

$\angle C = \angle C$ (reflexive)

Then

$\angle D = \angle B = 40°$ # (the third angle's theorem)

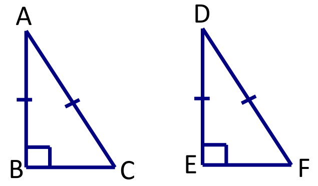
The reasoning of example 4 is the same as that of example 3. Now you can easily understand what I mean.

The following triangle congruence theorem is a special one. It can only be applied to right triangles.

RHS triangle congruence theorem:

If the hypotenuse and a leg of a right triangle are congruent to corresponding parts of another right triangle, these two right triangles are congruent.

As shown in the figure on the right,



In ΔABC and ΔDEF, $\angle E = \angle B = 90°$,

$\overline{AB} = \overline{DE}$, $\overline{AC} = \overline{DF}$

Then $\Delta ABC \cong \Delta DEF$

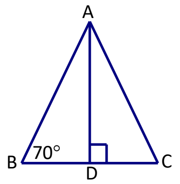
(triangle ABC is congruent to triangle DEF)

There is an example of the RHS triangle congruence theorem.

Ex 5:

In ΔABC, $\overline{AD}$ is perpendicular to $\overline{BC}$, $\overline{AB} = \overline{AC}$

$\angle B = 70°$, then $\angle C = ?$



Sol:

Since we are learning triangle congruency, we try to look for the congruent triangles.

In ΔABD and ΔACD,

$\overline{AB} = \overline{AC}$ (given)

$\angle ADB = \angle ADC = 90°$, ($\overline{AD}$ is perpendicular to $\overline{BC}$)

$\overline{AD} = \overline{AD}$ (reflexive)

Then

$\Delta ABD \cong \Delta ACD$ (RHS)

$\angle C = \angle B = 70°$# (CPCTC)

The five triangle congruence theorems above are essential to future reasoning. Please review them thoroughly till you can recite them without hesitating.

Before we finish this section, I want to raise a critical question.

Now we have learned these five triangle congruence theorems:

SSS, SAS, ASA, AAS, and RHS. Have you noticed that there are no combinations like AAA or ASS?

Please discuss this issue with your classmates and carefully consider it before continuing with your studies.

Now let's dive into this issue together.

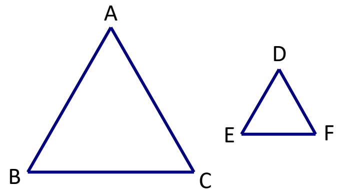
AAA (does not guarantee the triangle congruency)

When three interior angles of a triangle are congruent to three angles of another triangle, these two triangles are not guaranteed to be congruent.

(Remember I always say, if we want to point out that a math statement is wrong, the easiest way is to offer a counterexample.)

Counterexample:

Two equilateral triangles ΔABC and ΔDEF are shown on the right.



All the interior angles of these two equivalent triangles are 60°

Even though these two triangles look similar, obviously, they don't have the same size, they are not congruent.

So AAA doesn't guarantee the congruency of triangles.

Let's discuss the reason why ASS does not guarantee the triangle congruency either.

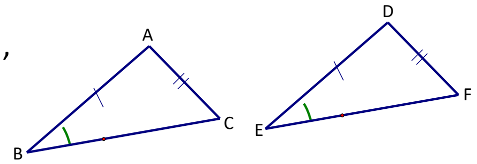
ASS (does not guarantee the triangle congruency)

If an angle and two adjacent sides in a row of a triangle are congruent to the corresponding parts of another triangle, these two triangles are not necessarily congruent.

In Figure 1, given ΔABC and ΔDEF, $\angle B=\angle E$,

$\overline{AB}=\overline{DE}$, and $\overline{AC}=\overline{DF}$.

Then these two triangles are identical.



(You can check by cutting down one triangle and putting it on top of the other triangle, you will find they are overlapped perfectly.)

Figure 1

In this case, the ASS looks promising to guarantee the triangle congruency.

But wait a second. Let me show you the different result we can get from the same given conditions.

Counterexample:

In Figure 2, given ΔABC and ΔDEF, $\angle B=\angle E$,

$\overline{AB}=\overline{DE}$, and $\overline{AC}=\overline{DF}$

Draw an arc with the radius $\overline{DF}$

from the center of D.

The arc intersects $\overline{EF}$ at points F and G.

Of course, $\overline{DF}=\overline{DG}$. (same radius)

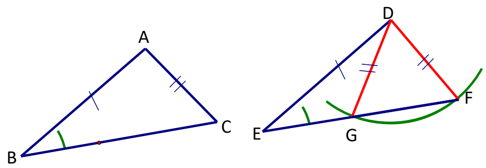


Figure 2

In ΔABC and ΔDEG, $\angle B=\angle E$,

$\overline{AB}=\overline{DE}$, and $\overline{AC}=\overline{DF}.=\overline{DG}$

But when we look at ΔABC and ΔDEG, they are definitely not congruent.

Therefore, ASS does not guarantee the congruency of triangles.

We say it's the Ambiguous case.

This is enough for this lesson. We will do more practice in the next class.

Reference:

教育部國民中學數學 108 課綱

教育部審定國民中學數學科南一、康軒以翰林及第五冊課本

製作者 北市金華國中 郝曉青