多項式的加減
Adding and Subtracting Polynomials

Class:__________ Name: __________

1. Introduction to polynomial

Algebraic expressions like $2x+3$, x^2-6, x^3+x^2-x+1, which consist of addition and multiplication operations involving numbers and variable x, are called polynomial of x. We can also say that polynomials are expressions of one term or the sum of more than one term that contain different powers of the same value. However, an algebraic expression that contains a term with a variable x appears in the denominator or within absolute value symbols, such as $\frac{1}{x^2}$, $|x|+5$, is NOT a polynomial of x.

Exercise 1

Which of the following expressions are polynomials?

□ $\frac{1}{x+7}$ □ $2x^2+\frac{x}{3}-3$ □ $-|4x-2|$ □ $7x^3$ □ $x+7-4x^2$

In polynomial $4x^2+5x+3$, each part separated by a plus sign is called a term of a polynomial, so $4x^2$, $5x$, and 3 are all terms of $4x^2+5x+3$.

◆ $4x^2$ is called the quadratic term (or x^2 term) of this polynomial, and 4 is the coefficient of the quadratic term (or x^2 term).
◆ $5x$ is called the linear term (or x term) of this polynomial, and 5 is the coefficient of the linear term (or x term).
◆ 3 is called the constant term of this polynomial.

When a polynomial contains minus signs, such as $7x^2-9x+4$, it can be written as $7x^2+(-9x)+4$. Therefore, the coefficient of the x^2 term is 7, the coefficient of the x term is -9 and the constant term is 4.

Terms with a coefficient of 0 are usually omitted. For example, $-9x^3+0x^2-6x+11$ is simplified to $-9x^3-6x+11$. Therefore, in the polynomial $-9x^3-6x+11$, we can consider the x^2 term as $0x^2$, so the coefficient of x^2 term is 0.

If the coefficient of a term is 1 and it is not a constant term, we also omit the coefficient. For example, $5x^2+1x+4$ is simplified to $5x^2+x+4$. Therefore, in polynomial $5x^2+x+4$, the coefficient of x term is 1.

Exercise 2

Find the coefficient of each term.

polynomial	$-2x^3+x^2+4x-11$	$4x-x^3+3$
coefficient of x^3 term		
coefficient of x^2 term		
coefficient of x term		
coefficient of constant term		

The degree of a polynomial is the greatest degree of its terms. In other words, the term with the greatest exponent determines the degree of the polynomial. For example, the degree of $4x^3+2x-1$ is 3 because the term with greatest degree is $4x^3$, and its degree is 3.

Exercise 3

Find the degree of polynomial (1) $2x^2-8x-7$ (2) $-15x+x^3$

2. Arranging the polynomial in ascending or descending order

We usually arrange the terms of a polynomial in order of the degrees of x. A polynomial is arranged in ascending order if the degree of each term increases from left to right. Conversely, a polynomial is arranged in descending order if the degree of each term decreases from left to right. For example, ascending order of the polynomial $5x-7x^3-2+8x^2$ is $-2+5x+8x^2-7x^3$, and descending order of $5x-7x^3-2+8x^2$ is $-7x^3+8x^2+5x-2$. In junior high school, if there are no specific requirements, we usually arrange the polynomials in descending order.

Exercise 4

Write down the polynomial $6x^2-3+7x$ in both ascending and descending order.

3. Like terms

In a polynomial, terms with the same variable raised to the same power are called like terms. For example, in polynomial $5x^2+4-2x^2+7x-9+8x$, $5x^2$ and $-2x^2$ are like terms; $7x$ and $8x$ are like terms; 4 and -9 are like terms. We can add or subtract polynomials by combining like terms.

Exercise 5

Which of the following expressions are like terms with $4x^2$?

☐ $7x$ ☐ $-0.3x^2$ ☐ $-85x^3$ ☐ $13x^2$

4. Adding polynomials

Example 1

Calculate $(4x+7x^2-3)+(5-8x+6x^2)$

[Method 1] Adding polynomials horizontally

When adding polynomials horizontally, we remove the parentheses and combine like terms.

$(4x+7x^2-3)+(5-8x+6x^2)$

$=4x+7x^2-3+5-8x+6x^2$

$=(7x^2+6x^2)+(4x-8x)+(-3+5)$

$=13x^2-4x+2$

[Method 2] Adding polynomials vertically

When adding polynomials vertically, we arrange the polynomials in descending order first, and then combine like terms by arranging like terms in columns.

$$\begin{array}{rr} & 7x^2+4x-3 \\ +) & 6x^2-8x+5 \\ \hline & 13x^2-4x+2 \end{array}$$

Exercise 6

Calculate $(3x^2-7x+4)+(-4x^2-x-9)$

Example 2

Calculate $(-8x^2-4+x)+(5x^2-6)$

[Method 1] Adding polynomials horizontally

$(-8x^2-4+x)+(5x^2-6)$

$=-8x^2-4+x+5x^2-6$

$=(-8x^2+5x^2)+x+(-4-6)$

$=-3x^2+x-10$

[Method 2] Adding polynomials vertically

If a term in one of the polynomial is missing, it means the coefficient of the term is 0. To align like terms, we can write the term with a coefficient of 0.

$$\begin{array}{rr} & -8x^2+x-4 \\ +) & 5x^2+0x-6 \\ \hline & -3x^2+x-10 \end{array}$$

Exercise 7

Calculate $(-4x^2+1)+(9-6x)$

5. Subtracting polynomials

$\boxed{\text{Example 3}}$

Calculate $(-4x+x^2+7)-(9x^2-5x-3)$

[Method 1] Subtracting polynomials horizontally

$(-4x+x^2+7)-(9x^2-5x-3)$

$=-4x+x^2+7-9x^2+5x+3$

$=(x^2-9x^2)+(-4x+5x)+(7+3)$

$=-8x^2+x+10$

[Method 2] Subtracting polynomials vertically

$$\begin{array}{lr} & x^2-4x+7 \\ -) & 9x^2-5x-3 \\ \hline & -8x^2+x+10 \end{array}$$

$\boxed{\text{Exercise 8}}$

Calculate $(-x^2-3x+8)-(-4+x-6x^2)$

一、設計理念：

1. 國外課本通常會直接說習慣上多項式會將次數由高排到低呈現，但國內課本會同時呈現升冪排列與降冪排列，且未說明升冪排列與降冪排列的使用時機。本課程中同時介紹升冪排列與降冪排列，但在最後有特別說明除非有特殊要求，否則國中階段多項式通常以降冪排列表示。

二、英文詞彙：

中文	英文
多項式	polynomial
項	term
二次項	quadratic term
一次項	linear term
常數項	constant term
係數	coefficient
次數	degree
升冪排列	ascending order
降冪排列	descending order
同類項	like terms
x^2	x squared / x to the power of two
x^3	x cubed / x to the power of three
橫式加法	add horizontally
直式加法	add vertically

三、教學參考範例：

<table>
<tr><td rowspan="2">1
【多項式的加法】
Adding polynomials</td><td>4. Adding polynomials
Example 1
Calculate $(4x+7x^2-3)+(5-8x+6x^2)$</td></tr>
<tr><td>We have learned basic knowledge about polynomials. Now, we are going to introduce how to add polynomials. There are two ways to add two polynomials - the horizontal method and the vertical method.</td></tr>
</table>

<table>
<tr><td rowspan="4">1
【多項式的加法】
Adding polynomials</td><td>[Method 1] Adding polynomials horizontally
When adding polynomials horizontally, we remove the parentheses and combine like terms.
$(4x+7x^2-3)+(5-8x+6x^2)$
$=4x+7x^2-3+5-8x+6x^2$
$=(7x^2+6x^2)+(4x-8x)+(-3+5)$
$=13x^2-4x+2$</td></tr>
<tr><td>Here, we learn the horizontal method first. We write down "$(4x+7x^2-3)+(5-8x+6x^2)$". Then, remove the parentheses. The first parenthesis can be removed directly because it is at the beginning of the operations. The second parenthesis can also be removed because there is a positive sign before the parenthesis. Therefore, we can write the original expressions as "$4x+7x^2-3+5-8x+6x^2$".
Next, we need to combine like terms. Like terms are terms that have the same variable raised to the same power, so $7x^2$ and $6x^2$ are like terms. Similarly, $4x$ and $-8x$ are like terms. -3 and 5 are like terms.
Let's start with the x^2 terms. Combining $7x^2$ and $6x^2$ together, we get $7x^2+6x^2=13x^2$. Then, move to the x terms. Combining $4x$ and $-8x$ together, we get $4x+(-8x)=-4x$. Finally, let's combine the constant term. Combining -3 and 5, and we have $-3+5=2$.
We get the simplified expression: "$13x^2-4x+2$". We can arrange the final answer in ascending order or descending order, so it's OK if you want to write it as "$2-4x+13x^2$".</td></tr>
<tr><td>[Method 2] Adding polynomials vertically
When adding polynomials vertically, we arrange the polynomials in descending order first, and then combine like terms by arranging like terms in columns.
$$\begin{array}{rr} & 7x^2+4x-3 \\ +) & 6x^2-8x+5 \\ \hline & 13x^2-4x+2 \end{array}$$</td></tr>
<tr><td>Now, let's look at the second method - the vertical method. When adding polynomials vertically, align like terms and then add them. It's similar to how to add numbers in elementary school.
In the beginning, we need to arrange the polynomial so that we can align the like terms later. We usually write polynomials in descending order, so the first polynomial can be written as "$7x^2+4x-3$", and the second polynomial can be written as "$6x^2-8x+5$".
Then, write down the polynomial vertically, and we can easily align the like terms. Starting with the x^2 term, we have $7x^2+6x^2=13x^2$. Next, for the x term, we have $4x+(-8x)=-4x$. For the constant terms, we have $-3+5=2$. Therefore, we get the final result is $13x^2-4x+2$.
Note that whether we add polynomials horizontally or vertically, we end up with the same result, so it is a good way to verify our answer by using both methods.</td></tr>
</table>

<table>
<tr><td rowspan="7">2
【多項式的減法】
Subtracting polynomials</td><td>5. Subtracting polynomials
Example 3
Calculate $(-4x+x^2+7)-(9x^2-5x-3)$</td></tr>
<tr><td>Now that we know how to add polynomials, we are going to learn how to subtract polynomials. This problem is similar to the addition problem we just did, but there are some differences to keep in mind.</td></tr>
<tr><td>[Method 1] Subtracting polynomials horizontally
$(-4x+x^2+7)-(9x^2-5x-3)$
$=-4x+x^2+7-9x^2+5x+3$
$=(x^2-9x^2)+(-4x+5x)+(7+3)$
$=-8x^2+x+10$</td></tr>
<tr><td>The problem we are working with is "$(-4x+x^2+7)-(9x^2-5x-3)$". Let's start with the horizontal method. First, we need to remove the parentheses. As in the past two problems, the first parenthesis can be moved directly. However, the sign before the second parenthesis is negative, so we have to apply the distributive property. Notice that by distributive property, the positive in the parenthesis becomes negative, and the negative in the parenthesis becomes positive. After removing the parenthesis, the original expressions equal "$-4x+x^2+7-9x^2+5x+3$."
Then, we combine like terms. In the beginning, combine x^2 terms, we have $x^2-9x^2=-8x^2$. Next, combine x terms, we have $-4x+5x=x$. Finally, combine the constant terms, we have $7+3=10$. Putting it all together, we get the simplified expression $-8x^2+x+10$, and it is our final answer.</td></tr>
<tr><td>[Method 2] Subtracting polynomials vertically
$$\begin{array}{rr} & x^2-4x+7 \\ -) & 9x^2-5x-3 \\ \hline & -8x^2+x+10 \end{array}$$</td></tr>
<tr><td>Now, let's look at the vertical method for subtraction. This method also involves aligning the like terms in columns, just like addition, but we need to be careful with the negative sign.
First, by arranging the polynomials in descending order and aligning the like terms, we have "x^2-4x+7" in the first row, and "$9x^2-5x-3$" in the second row.
Then, we subtract vertically by combining like terms in each column. Starting with the x^2 terms, we have $x^2-9x^2=-8x^2$. Next, for the x terms, we have $-4x-(-5x)=x$. For the constant terms, we have $7-(-3)=10$. Be careful when subtracting like terms. Students usually get in trouble at this step. If we do it correctly, we get the final result is $-8x^2+x+10$, which is also the same as the result in the horizontal method.</td></tr>
</table>

製作者：臺北市雙園國中 劉家宇