

Topic: The equation of a hyperbola in standard form

1. The center is at (0,0) and the foci lie on the x-axis

(a) Find the equation of a hyperbola in standard form with foci (5,0) and (-5,0), P is any point on the hyperbola satisfied $|\overline{PF_1} - \overline{PF_2}| = 6$.

(b) Find the equation of a hyperbola in standard form with foci (c,0) and (-c,0), P is any point on the hyperbola satisfied $|\overline{PF_1} - \overline{PF_2}| = 2a$.

(c) Exercise

Identify the values of a, b, and c on the following hyperbola, then find the center, vertices, foci, the length of the transverse axis, and conjugate axis. Then sketch the hyperbola.

$$(1) \frac{x^2}{36} - \frac{y^2}{64} = 1$$

$$(2) \frac{x^2}{64} - \frac{y^2}{36} = 1$$

$$(3) x^2 - y^2 = 36$$

$$(4) 9x^2 - 16y^2 = 144$$

$$(5) |\sqrt{(x-2)^2 + y^2} - \sqrt{(x+2)^2 + y^2}| = 2$$

2. The center is at (0,0) and the foci lie on the y-axis

(a) Find the equation of a hyperbola in standard form with foci (0,5) and (0,-5), P is any point on the hyperbola satisfied $|\overline{PF}_1 - \overline{PF}_2| = 6$.

(b) Find the equation of a hyperbola in standard form with foci (0,c) and (0,-c), P is any point on the hyperbola satisfied $|\overline{PF}_1 - \overline{PF}_2| = 2a$.

(c) Exercise

Identify the values of a, b, and c on the following hyperbola, then find the center, vertices, foci, the length of the transverse axis, and conjugate axis. Then sketch the hyperbola.

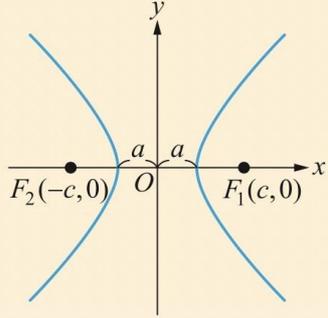
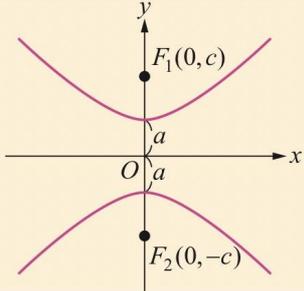
$$(1) -\frac{x^2}{25} + \frac{y^2}{144} = 1$$

$$(2) \frac{y^2}{64} - \frac{x^2}{36} = 1$$

$$(3) 9x^2 - 4y^2 = -36$$

$$(4) |\sqrt{x^2 + (y - 5)^2} - \sqrt{x^2 + (y + 5)^2}| = 8$$

3. The conclusion

Standard form	Foci	Graph
	$(c,0)$ and $(-c,0)$	
	$(0,c)$ and $(0,-c)$	

4. Challenge

Find the equation of the hyperbola in standard form with foci $(-4,2)$, $(2,2)$, and the transverse axis of length 4.

(Hint: use the concept of translation)

Topic: The equation of a hyperbola in standard form 使用建議

Warm-up:

[教學活動安排]

從中心在(0,0)且數據簡單的雙曲線來帶領學生一步步從幾何定義寫出式子化簡後得到標準式，接著推導一般式及練習。

[可參考英文開場\提問]

If we are given certain information about a hyperbola, how do we find the equation of a hyperbola?

Let's see this example 1(a).

Like what we did with parabolas and ellipses.

What did we do to find the equation?

Can any volunteers who can help us to recall what we did before?

1. The center is at (0,0) and the foci lie on the x-axis

(a) Find the equation of a hyperbola in standard form with foci (5,0) and (-5,0), P is any point on the hyperbola satisfied $|\overline{PF}_1 - \overline{PF}_2| = 6$.

P(x,y) is a point on the hyperbola. Then, by the definition.

$ \overline{PF}_1 - \overline{PF}_2 = 6$	The absolute value of the difference of the distance P to F_1 and F_2 is six.
$ \sqrt{(x-5)^2 + y^2} - \sqrt{(x+5)^2 + y^2} = 6$	Apply the distance formula to obtain the equation.(英文實在太長了，建議不要念)
$(\sqrt{(x-5)^2 + y^2})^2 = (\pm 6 + \sqrt{(x+5)^2 + y^2})^2$	Rewrite the equation and then square both sides.
$-20x - 36 = \pm 12\sqrt{(x+5)^2 + y^2}$	Expand and simplify. We will get negative twenty x minus thirty six equals positive and minus twelve times the square root of x plus five squared plus y squared.

$256x^2 - 144y^2 = 2304$	<p>Square both sides and simplify.</p> <p>We will get two hundred fifty six x squared minus one hundred forty four y squared equals two thousand three hundred and four.</p>
$\frac{x^2}{9} - \frac{y^2}{16} = 1$	<p>Divide both sides by two thousand three hundred and four.</p> <p>We will get x squared over nine minus y squared over sixteen equals one.</p> <p>We say this equation is the standard form of the hyperbola.</p>

Take a closer look at the coefficients of this standard form, what have you noticed?

What will the standard form become if we change the focus and the length of the transverse axis?

Let's look at (b)

- (b) Find the equation of a hyperbola in standard form with foci $(c,0)$ and $(-c,0)$, P is any point on the hyperbola satisfied $|\overline{PF_1} - \overline{PF_2}| = 2a$.

P(x,y) is a point on the hyperbola.

$$|\sqrt{(x-c)^2 + y^2} - \sqrt{(x+c)^2 + y^2}| = 2a$$

Rewrite, square both sides, simplify, square both sides again, expand and simplify, and reduces to

$$(c^2 - a^2)x^2 - a^2y^2 = a^2(c^2 - a^2)$$

We know that $b^2 = c^2 - a^2$

So the equation becomes $b^2x^2 - a^2y^2 = a^2b^2$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Now we can use this conclusion from (b) to quickly sketch a hyperbola from an equation.

See exercise(c)

(c) Exercise

Identify the values of a, b, and c on the following hyperbola, then find the center, vertices, foci, the length of the transverse axis, and conjugate axis. Then sketch the hyperbola.

$$(1) \frac{x^2}{36} - \frac{y^2}{64} = 1$$

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What if the transverse axis is vertical? How will the standard form change?

Let's look at this example. Have a go!

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$ \overline{PF_1} - \overline{PF_2} = 6$	<p>The absolute value of the difference of the distance P to F_1 and F_2 is six.</p>
$ \sqrt{x^2 + (y - 5)^2} - \sqrt{x^2 + (y + 5)^2} = 6$	<p>Apply the distance formula to obtain the equation.(英文實在太長了，建議不要念)</p>
$(\sqrt{x^2 + (y - 5)^2})^2 = (\pm 6 + \sqrt{x^2 + (y + 5)^2})^2$	<p>Rewrite the equation and then square both sides.</p>
$-20y - 36 = \pm 12\sqrt{x^2 + (y + 5)^2}$	<p>Expand and simplify. We will get negative twenty y minus thirty six equals positive and minus twelve times the square root of x squared plus y plus five squared.</p>
$256y^2 - 144x^2 = 2304$	<p>Square both sides and simplify. We will get two hundred fifty six y squared minus one hundred forty four x squared equals two thousand three hundred and four.</p>
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Rewrite, square both sides, simplify, square both sides again, expand and simplify, and reduces to

$$(c^2 - a^2)y^2 - a^2x^2 = a^2(c^2 - a^2)$$

We know that $b^2 = c^2 - a^2$

So the equation becomes $b^2y^2 - a^2x^2 = a^2b^2$

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

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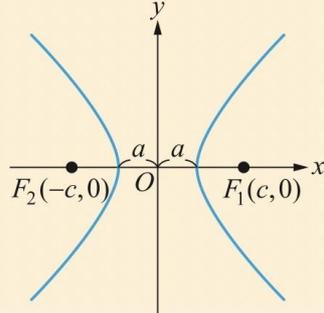
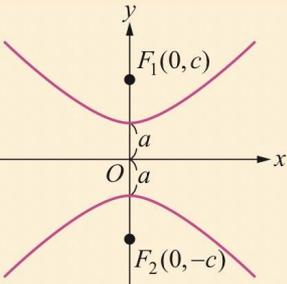
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Let's sum up what we've learned. Complete the following form.

3. The conclusion

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